Instructions:

1. PLEASE SHOW ALL WORK!

2. **The exam is due at 5:00 PM on Tuesday, May 16.** Early submissions are gratefully accepted.

3. Unless the directions for a particular problem say otherwise, you may use Mathematica, Maple, MATLAB, or any other resource.

4. Use of the Integral Table is permitted, but please indicate which formula(s) you use.

5. Have a great summer. It has been a pleasure to work with you this year.

1. (10 pts.) Find the area of the region in the $xy$ plane bounded by the graphs of $y = 1 - x^2$ and $y = x^4 - 1$.

2. (10 pts.) Consider the region in the $xy$ plane bounded by the $y$ axis, the line $y = 1$, and the curve $y = \sqrt{x}$. Find the volume of the solid obtained by rotating this region about the $x$ axis.

3. (15 pts.) Evaluate the following three definite integrals. You may use Mathematica to check your work, but please do these problems by hand.

   a) $\int_1^4 \sqrt{x} \ln(x) \, dx$
   b) $\int_1^4 \frac{\sin(\sqrt{x})}{\sqrt{x}} \, dx$
   c) $\int_2^4 \frac{16}{x^2(x^2 + 4)} \, dx$

4. (10 pts.) Consider the integral $\int_0^{\pi/2} e^{-\sin(x)} \, dx$ If you wanted to approximate this integral with an error no larger than $10^{-4}$ using the Trapezoid Rule, how many intervals would you need? Explain.

5. (15 pts.) Determine whether each of the following improper integrals converges or diverges. Explain. (“Mathematica said so” is not a sufficient explanation.)

   a) $\int_1^\infty \frac{1}{x + \sqrt{x}} \, dx$
   b) $\int_0^1 \frac{1}{\sqrt{1-x}} \, dx$

OVER
6. (10 pts.) Determine whether each of the following series converges or diverges. Be sure to explain what test you are using.

a) \( \sum_{k=0}^{\infty} \frac{2^k}{k!} \)

b) \( \sum_{k=1}^{\infty} \frac{k}{k^2 + 1} \)

7. (15 pts.) Consider the power series \( \sum_{k=1}^{\infty} \frac{(x + 1)^k}{k^2} \)

a. Find the radius of convergence of the given series.

b. Find the interval of convergence of the given series.

8. (15 pts.) Let \( f(x) = \sinh(x) \).

a. Show that the Maclaurin series for \( f \) is \( \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} \).

b. Find the radius of convergence of the series in part a.

c. Approximate \( \sinh(0.5) \) with an error no larger than \( 10^{-4} \) using a partial sum of the Maclaurin series in part a. Be sure to give the approximate value AND explain how you know your error is no greater than \( 10^{-4} \). (“Mathematica said so” is not a sufficient explanation.)

Hint: \( \sinh(x) \leq \cosh(x) \) for all \( x \) and \( \cosh(0.5) = \frac{1}{2} \left( e^{0.5} + e^{-0.5} \right) < \frac{1}{2} \left( e^{0.5} + e^{0.5} \right) = e < \sqrt{4} = 2 \).

Extra credit problem (5 pts.) Fill in the blank:

If the only thing I remember from Calculus II is that a definite integral is _____ , I will have earned 5 extra credit points on this exam and Professor Pennell will die happy.