92.231 Calculus III Spring 2004
Review Sheet for Exam #1
**Necessary Skills**

| Section | You should be able to |
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| 9.1 | • compute the distance between given points in 3-space<br>• find the equation of a sphere given a description of the sphere<br>• find the center and radius of a sphere given the equation of the sphere |
| 9.2 | • compute sums and scalar multiples of given vectors<br>• find the magnitude of a given vector<br>• find the components of a vector joining two given points<br>• find a unit vector in the direction of a given vector<br>• solve physical problems involving force and velocity using vectors |
| 9.3 | • compute the dot product of two given vectors<br>• use dot products to find the angle between two vectors<br>• use dot products to determine whether two vectors are orthogonal<br>• compute the projection of one vector onto another vector<br>• compute the work done by a specified force on an object moving along the line joining two given points |
| 9.4 | • compute the cross product of two given vectors<br>• use the properties of the cross product to solve such problems as computing the area of a parallelogram, computing the volume of a parallepiped<br>• find a vector orthogonal to two given vectors |
| 9.5 | • find the equation of a plane given a description of the plane<br>• find the parametric, vector, or symmetric equations of a line, given a description of the line |
| 10.1 | • find the domain of a given vector function<br>• determine whether a given point lies on a curve with given parametric representation |
| 10.2 | • find the derivative of a given vector-valued function<br>• find the integral of a given vector-valued function<br>• find the unit tangent vector to a curve given the position vector<br>• find the line tangent to a given curve at a given point |
| 10.3 | • find the unit tangent vector, unit normal vector, and curvature of a curve given the position vector<br>• compute the length of a given curve |
| 10.4 | • compute velocity, speed, and acceleration of an object given its position vector<br>• compute position, velocity, and speed of an object given its acceleration vector |

**Answers to Practice Exam Questions**

1. a) $\langle -1, 1, 6\rangle$ b) $-2$ c) $\langle 4, -14, -3\rangle$ d) $\langle -4/9, -2/9, 4/9\rangle$
2. $2x + y - 2z = -1$. 3. a) 3 b) Yes, because the distance from $P$ to $Q$ is 3.
3. c) $(x-1)^2 + (y+1)^2 + (z-2)^2 = 9$ 4. b) $3e^2 - 3 \approx 19.167$
5. a) $\langle 2t, 1, -1\rangle$ b) $\mathbf{r}(2) = \langle 4, 2, 0\rangle$ c) $x = 4 + 4t, y = 2 + t, z = -t$
6. a) $\left\langle \frac{4}{5}\cos(4t), \frac{3}{5}, \frac{4}{5}\sin(4t) \right\rangle$ b) $\langle -\sin(4t), 0, \cos(4t)\rangle$ c) $16/25$

Calculus III Practice Exam #1 Spring 2004

**There is no guarantee that the actual exam will bear any resemblance to this practice exam. The purpose of the practice exam is to give you an idea of the approximate length and the type of problem that you can expect on the actual exam.**

**Problem #1 (20 points)**
Let $\mathbf{a} = \langle 2, 1, -2 \rangle$ and let $\mathbf{b} = \langle 3, 0, 4 \rangle$.

a. Compute $\mathbf{a} - \mathbf{b}$.

b. Compute $\mathbf{a} \cdot \mathbf{b}$.

c. Compute $\mathbf{a} \times \mathbf{b}$.

d. Compute $\text{proj}_{\mathbf{a}}\mathbf{b}$ (the vector projection of $\mathbf{b}$ onto $\mathbf{a}$).

**Problem #2 (10 points)**
Find the equation of the plane containing the point $(0, 3, 2)$ and perpendicular to the vector $\langle 2, 1, -2 \rangle$.

**Problem #3 (15 points)**
Let $P$ denote the point $(1, -1, 2)$ and let $S$ denote the sphere with center at $P$ and radius 3.

a. Let $Q$ denote the point $(3, 0, 0)$. Compute the distance from $P$ to $Q$.

b. Does $Q$ lie on the sphere $S$? Why or why not?

c. Find the equation of the sphere $S$.

**Problem #4 (15 points)**
Let $C$ denote the curve described by the vector function $\mathbf{r}(t) = \left\langle e^{2t}, 2e^{2t}, 2e^{2t} \right\rangle$, $0 \leq t \leq 1$.

a. Show that $|\mathbf{r}'(t)| = 6e^{2t}$.

b. Find the length of $C$.

**Problem #5 (20 points)**
Let $C$ denote the curve described by the vector function $\mathbf{r}(t) = \langle t^2, t, 2 - t \rangle$

a. Find $\mathbf{r}'(t)$.

b. Show that the point $(4, 2, 0)$ lies on $C$.

c. Find parametric equations for the line tangent to $C$ at the point $(4, 2, 0)$.

**Problem #6 (20 points)**
Let $C$ denote the curve described by the vector function $\mathbf{r}(t) = \langle 1 + \sin(4t), 3t, 2 - \cos(4t) \rangle$.

a. Find the unit tangent vector $\mathbf{T}(t)$.

b. Find the unit normal vector $\mathbf{N}(t)$.

c. Find the curvature $\kappa$.