

Necessary Skills

Section	You should
9.6	<ul style="list-style-type: none"> be able to sketch cylinders and quadric surfaces be able to find and sketch the domain of a given function
9.7	<ul style="list-style-type: none"> be able to find the cylindrical coordinates of a point given its Cartesian coordinates be able to find the Cartesian coordinates of a point given its cylindrical coordinates be able to find the spherical coordinates of a point given its Cartesian coordinates be able to find the Cartesian coordinates of a point given its spherical coordinates be able to describe surfaces whose equations are given in cylindrical or spherical coordinates be able to write the equation of a surface in cylindrical or spherical coordinates, given its equation in Cartesian coordinates
10.5	<ul style="list-style-type: none"> be able to write parametric equations for a given surface
11.1	<ul style="list-style-type: none"> be able to compute the value of a given function at a given point be able to sketch the graph of a given function of 2 variables be able to find the domain and range of a given function of 2 or 3 variables be able to draw a contour map of a given function of 2 variables be able to describe the level surfaces of a given function of 3 variables
11.3	<ul style="list-style-type: none"> be able to find partial derivatives (of any order) of a given function
11.4	<ul style="list-style-type: none"> be able to find the tangent plane to a given surface at a given point be able to find the linear approximation to a given function at a given point and use the linear approximation to compute approximate values of the function be able to find the differential of a given function of 2 or 3 variables and use the differential to approximate increment of the function
11.5	<ul style="list-style-type: none"> be able to apply the chain rule to functions of several variables
11.6	<ul style="list-style-type: none"> be able to find the gradient of a given function be able to find the directional derivative of a given function at a given point in a given direction know that the gradient vector points in the direction of most rapid increase of a function know that the magnitude of the gradient vector is the maximum rate of increase of a function know that the gradient vector is perpendicular to level curves/surfaces
11.7	<ul style="list-style-type: none"> be able to find local maxima, minima, and saddle points of a given function of 2 variables
12.1	<ul style="list-style-type: none"> represent the volume of a solid as a double integral approximate a double integral over a rectangular region by computing a Riemann sum
12.2	<ul style="list-style-type: none"> evaluate an iterated integral evaluate a double integral over a rectangle by representing it as an iterated integral

Answers to Practice Exam Questions

- $\frac{\partial z}{\partial x} = y \cos(xy) + 2xy^3$ $\frac{\partial^2 z}{\partial x^2} = -y^2 \sin(xy) + 2y^3$ $\frac{\partial^2 z}{\partial y \partial x} = \cos(xy) - xy \sin(xy) + 6xy^2$.
- 28/5 3. a) $\langle -2, 7, -1 \rangle$ b) $3\sqrt{6}$
- $500\pi \text{ cm}^3/\text{sec}$ 5. f has a saddle point at $(0, 0)$ and a local minimum at $(3, 9)$.
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There is no guarantee that the actual exam will bear any resemblance to this practice exam. The purpose of the practice exam is to give you an idea of the approximate length and the type of problem that you can expect on the actual exam.

Problem #1 (10 points)

Let $z = \sin(xy) + x^2y^3$. Find $\frac{\partial z}{\partial x}$, $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial y \partial x}$.

Problem #2 (20 points)

Let $f(x, y) = x^2 - y^4$, let P denote the point $(2, 1)$, and let $\mathbf{v} = \langle 4, -3 \rangle$. Find the directional derivative of f at P in the direction of vector \mathbf{v} .

Problem #3 (15 points)

Let $f(x, y, z) = x^2y + y^3z$ and let P denote the point $(1, -1, 2)$.

- Find a vector in the direction in which f increases most rapidly at P .
- Find the maximum rate of increase of f at P .

Problem #4 (20 points)

The radius r and the height h of a right circular cylinder change with time. At a certain instant the dimensions are $r = 10$ cm and $h = 20$ cm, the radius r is increasing at a rate of 2 cm/sec, and the height h is decreasing at a rate of 3 cm/sec. At what rate is the volume V of the cylinder changing at that instant? (Hint: $V = \pi r^2 h$.)

Problem #5 (20 points)

Let $f(x, y) = x^3 - 3xy + \frac{1}{2}y^2$.

- Show that $(0, 0)$ and $(3, 9)$ are the only critical points of f .
- Determine whether f has a local max, a local min, or a saddle point at $(0, 0)$.
- Determine whether f has a local max, a local min, or a saddle point at $(3, 9)$.

Problem #6 (15 points)

Find the volume of the solid lying under the circular paraboloid $z = x^2 + y^2$ and above the rectangle $R = [-2, 2] \times [-3, 3]$.