92.231 Calculus III Spring 2004
Review Sheet for Final Exam

Major Concepts

- Partial derivative.
- Directional derivative.
- Linear approximation.
- Double integral. (Evaluate using double iterated integral in rectangular or polar coordinates.)
- Triple integral. (Evaluate using triple iterated integral in rectangular, cylindrical, or spherical coordinates.)
- Line integral. (Evaluate by expressing line integral as an ordinary definite integral.)
- Surface integral. (Evaluate by expressing surface integral as a double integral.)

Major Theorems

- Green's Theorem - relates a double integral over a region D to a line integral over the curve bounding D.
- Stokes's Theorem - relates a surface integral over a region S to a line integral over the curve bounding S.
- Divergence Theorem - relates a triple integral over a region E to a surface integral over the surface bounding E.

Types of Functions

- Scalar-valued function of 2 variables $f(x, y)$ - graph is a surface.
- Scalar-valued function of 3 variables $f(x, y, z)$ - also known as a scalar field.
- Vector-valued function of 1 variable $\mathbf{r}(t)$ - graph is a curve.
- Vector-valued function of 3 variables $\mathbf{F}(x, y, z)$ - also known as a vector field.

Important Operations

- Dot product of 2 vectors.
- Cross product of 2 vectors.
- Gradient of a scalar field: $\nabla f(x, y, z)$.
- Divergence of a vector field: $\nabla \cdot \mathbf{F}(x, y, z)$.
- Curl of a vector field: $\nabla \times \mathbf{F}(x, y, z)$.

Necessary Skills

Section	You should be able to
9.1	• compute the distance between given points in 3-space • find the equation of a sphere given a description of the sphere • find the center and radius of a sphere given the equation of the sphere
9.2	• compute sums and scalar multiples of given vectors • find the magnitude of a given vector • find the components of a vector joining two given points • find a unit vector in the direction of a given vector • solve physical problems involving force and velocity using vectors
9.3	• compute the dot product of two given vectors • use dot products to find the angle between two vectors • use dot products to determine whether two vectors are orthogonal • compute the projection of one vector onto another vector • compute the work done by a specified force on an object moving along the line joining two given points
9.4	• compute the cross product of two given vectors • use the properties of the cross product to solve such problems as computing the area of a parallelogram, computing the volume of a parallepiped • find a vector orthogonal to two given vectors
9.5	• find the equation of a plane given a description of the plane • find the parametric, vector, or symmetric equations of a line, given a description of the line
9.6	• be able to sketch cylinders and quadric surfaces • be able to find and sketch the domain of a given function
9.7	• be able to find the cylindrical coordinates of a point given its Cartesian coordinates • be able to find the Cartesian coordinates of a point given its cylindrical coordinates • be able to find the spherical coordinates of a point given its Cartesian coordinates • be able to find the Cartesian coordinates of a point given its spherical coordinates • be able to describe surfaces whose equations are given in cylindrical or spherical coordinates • be able to write the equation of a surface in cylindrical or spherical coordinates, given its equation in Cartesian coordinates
10.1	• find the domain of a given vector function • determine whether a given point lies on a curve with given parametric representation
10.2	• find the derivative of a given vector-valued function • find the integral of a given vector-valued function • find the unit tangent vector to a curve given the position vector • find the line tangent to a given curve at a given point
10.3	• find the unit tangent vector, unit normal vector, and curvature of a curve given the position vector • compute the length of a given curve
10.4	• compute velocity, speed, and acceleration of an object given its position vector • compute position, velocity, and speed of an object given its acceleration vector
10.5	• be able to write parametric equations for a given surface

Section	You should be able to
11.1	• be able to compute the value of a given function at a given point • be able to sketch the graph of a given function of 2 variables • be able to find the domain and range of a given function of 2 or 3 variables • be able to draw a contour map of a given function of 2 variables • be able to describe the level surfaces of a given function of 3 variables
11.3	• be able to find partial derivatives (of any order) of a given function
11.4	• be able to find the tangent plane to a given surface at a given point • be able to find the linear approximation to a given function at a given point and use the linear approximation to compute approximate values of the function • be able to find the differential of a given function of 2 or 3 variables and use the differential to approximate increment of the function
11.5	• be able to apply the chain rule to functions of several variables
11.6	• be able to find the gradient of a given function • be able to find the directional derivative of a given function at a given point in a given direction • know that the gradient vector points in the direction of most rapid increase of a function • know that the magnitude of the gradient vector is the maximum rate of increase of a function • know that the gradient vector is perpendicular to level curves/surfaces
12.1	• represent the volume of a solid as a double integral • approximate a double integral over a rectangular region by computing a Riemann sum
12.2	• evaluate an iterated integral • evaluate a double integral over a rectangle by representing it as an iterated integral
12.3	• represent the volume of a solid as a double integral • evaluate a double integral over a nonrectangular region by representing it as an iterated integral • sketch the region of integration and change the order of integration of a given iterated integral
12.4	• evaluate a double integral by representing it as an iterated integral using polar coordinates
12.5	• find the mass, center of mass, and moments of inertia of a given region in the xy plane, given the density function ρ
12.6	• find the area of a given surface
12.7	• evaluate a triple integral by representing it as an iterated integral in Cartesian coordinates
12.8	• evaluate a triple integral by representing it as an iterated integral in cylindrical coordinates • evaluate a triple integral by representing it as an iterated integral in spherical coordinates

Section	You should be able to
13.1	• sketch a given two-dimensional vector field • find the gradient of a given scalar function
13.2	• evaluate the line integral of a given function along a given curve • calculate the work done by a given force on an object moving along a given curve
13.3	• determine whether a given 2D vector field is conservative • find the potential function f of a given conservative vector field and use the potential function to evaluate a line integral
13.4	• use Green's Theorem to evaluate a line integral around a closed curve in the plane by converting it to a double integral, and vice versa
13.5	• find the divergence and curl of a given vector field • determine whether a given 3D vector field is conservative • find the potential function f of a given conservative vector field and use the potential function to evaluate a line integral
13.6	• evaluate a surface integral by representing it as an iterated integral
13.7	• use Stokes's Theorem to evaluate a line integral by converting it to a surface integral, and vice versa
13.8	• use Gauss's Divergence Theorem to evaluate a surface integral over a closed surface by converting it to a triple integral, and vice versa

Answers to Practice Exam Questions

1. a) $\mathbf{r}'(t) = \langle \cos(t), 2, -\sin(t) \rangle$, $\mathbf{r}''(t) = \langle -\sin(t), 0, -\cos(t) \rangle$
b) $\langle 1, 2, 0 \rangle$ or a nonzero multiple   c) $x = t,\ y = 2t,\ z = 1$ or an equivalent set of equations.
2. b) $x + 2y - z = 2$
3. -8
4. a) 1   b) $\langle 1, 1, 0 \rangle$
5. $1/3$
6. b) π
7. 0
8. a) 0   b) $\mathbf{0}$ d) $f(x, y, z) = xyz + xy + z + c$
9. b) $243\pi/2$
10. 0

Calculus III   Practice Final Exam   Spring 2003

There is no guarantee that the actual exam will bear any resemblance to this practice exam. The purpose of the practice exam is to give you an idea of the approximate length and the type of problem that you can expect on the actual exam.

1. Let C be the curve described by the vector function $\mathbf{r}(t) = \langle \sin(t), 2t, \cos(t) \rangle$.

   a. Find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

   b. Find a vector tangent to C at the point $(0, 0, 1)$.

   c. Find parametric equations of the line tangent to C at the point $(0, 0, 1)$.

2. Let L_1 be the line given by the parametric equations $x = 1 + t,\ y = 1 + t,\ z = 1 + 3t$ and let L_2 be the line given by the parametric equations $x = 2 + t,\ y = -t,\ z = -t$.

   a. Show that the point $(1, 1, 1)$ lies on both L_1 and L_2.

   b. Find the equation of the plane containing L_1 and L_2.

3. Suppose $z = f(x, y)$, $x = g(t)$, and $y = h(t)$. Find $\left.\dfrac{dz}{dt}\right|_{t=1}$ if $g(1) = 3,\ h(1) = 2,$ $g'(1) = -1,\ h'(1) = 2,\ f_x(3, 2) = 4,$ and $f_y(3, 2) = -2$.

4. Let $f(x, y, z) = x + y\cos(z)$, let P denote the point $(1, 2, 0)$, and let $\mathbf{a} = \langle 2, 1, -2 \rangle$.

   a. Find the directional derivative of f at P in the direction $\mathbf{a}$.

   b. Find a vector in the direction in which f increases most rapidly at P.

5. Evaluate $\displaystyle\iint_D (x^2 + y^2)\ dA$, where D is the triangular region with vertices $(-1, 0)$, $(0, 1)$ and $(1, 0)$.

6. Let $\mathbf{F}(x, y, z) = \langle 2xy, x^2 + 2yz, y^2 \rangle$ and let $f(x, y, z) = x^2y + y^2z$.

   a. Show that f is a potential for $\mathbf{F}$.

   b. Evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the helix $x = \sin(t),\ y = \cos(t),\ z = t,\ 0 \le t \le \pi$.

7. Evaluate the integral $\displaystyle\iiint_E z\ dV,$

   where E is the region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$.

8. Let $\mathbf{F}(x, y, z) = \langle yz + y, xz + x, xy + 1 \rangle$.

   a. Find $\nabla \cdot \mathbf{F}\ \ (= \mathrm{div}\,(\mathbf{F}))$

   b. Find $\nabla \times \mathbf{F}\ \ (= \mathrm{curl}\,(\mathbf{F}))$

   c. Show that $\mathbf{F}$ is conservative.

   d. Find a potential for $\mathbf{F}$.

9. Let S denote the part of the surface $z = 9 - x^2 - y^2$ above the xy plane and let $\mathbf{F} = \langle x, y, z \rangle$.

   a. Find a vector perpendicular to S at the point (x, y, z) having positive $\mathbf{k}$ component. (You should get $\langle 2x, 2y, 1 \rangle$ or a positive multiple of this.)

   b. Evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.

10. Let $\mathbf{F}(x, y, z) = \langle xz, yz, xy \rangle$ and let S denote the hemisphere $x^2 + y^2 + z^2 = 1,\ z \geq 0$.
Use Stokes's Theorem to evaluate $\displaystyle\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$, where $\mathbf{n}$ is the unit outer normal vector.