Due Date: Friday, May 7.

Please show all your work to receive full credit.

The work you turn in must be your own. Please do not talk to anyone except me about this exam.

Problem #1 (20 points)
Find the general solution to the fourth order d.e. \( y^{(4)} - 8y'' + 16y = 32t^2. \)

Please do not use MATLAB to solve this problem. Please solve the problem by hand and show all your work.

Problem #2 (20 points)
Use the Laplace Transform method to solve the following initial value problem:
\[ y'' - 5y' + 6y = 3e^{2t}, \quad y(0) = 0, \quad y'(0) = -3. \]

Please do not use MATLAB to solve this problem. Please solve the problem by hand and show all your work.

Problem #3 (25 points)
We have used the Laplace transform to solve single equations, but it can also be used to solve linear systems with constant coefficients. Consider the following system of differential equations subject to the initial conditions \( x(0) = 1, y(0) = 0: \)
\[
\begin{cases}
 x' = y \\
 y' = -x
\end{cases}
\]

a. Take the Laplace transform of each equation in the above system to obtain an algebraic system of equations for \( L\{x\} \) and \( L\{y\}. \)

b. Solve the system in part a for \( L\{x\} \) and \( L\{y\}. \)

c. Use your answer to part b to find \( x \) and \( y. \)

Please do not use MATLAB to solve this problem. Please solve the problem by hand and show all your work.

Problem #4 (10 points)
Use the definition of convolution to show that \((f * g)(t) = (g * f)(t)\).

Problem #5 (25 points)
Use the Laplace Transform to solve the integro-differential equation
\[ y'(t) + \int_0^t e^{-2(t-\tau)}y(\tau) \, d\tau = t, \quad y(0) = 1. \]

You might want to use MATLAB to find the inverse transform you need in the last step.