Name: _____

Consider the initial value problem

$$\begin{cases} \dfrac{dy}{dx} - y = 2\cos(x) \\[2mm] y(0) = -2 \end{cases}$$

1. Solve the given initial value problem analytically either by hand or by using MATLAB. To use MATLAB, open a mupad notebook using the command

   `>>mupad`

   In the mupad notebook, enter the command

   `yexact:=solve(ode({y'(x) - y(x) = 2*cos(x), y(0) = -2}, y(x)))`

2. Compute the value of the solution at $x = 1$ and enter it in the box labeled A in the table at the bottom of the page. Compute the value of the solution at $x = 2$ and enter it in the box labeled B. If you use your calculator to carry out these computations, make sure the calculator is in radian mode. If you want to use MATLAB for this calculation, use these commands in the mupad notebook

   `float(yexact | x=1)`

   `float(yexact | x=2)`

3. Use MATLAB for this part). Use the Runge-Kutta Method to generate an approximate solution to the initial value problem on the interval $0 \le x \le 2$ using $n = 2$ subintervals. Put the calculated y value corresponding to $x = 1$ in the box labeled C in the table, and put the calculated y value corresponding to $x = 2$ in the box labeled D. **Please show at least 4 digits after the decimal point.**

4. Use MATLAB for this part). Use the Runge-Kutta Method to generate an approximate solution to the initial value problem on the interval $0 \le x \le 2$ using $n = 4$ subintervals. Put the calculated y value corresponding to $x = 1$ in the box labeled E in the table, and put the calculated y value corresponding to $x = 2$ in the box labeled F. **Please show at least 4 digits after the decimal point.**

5. Fill in the rest of table by calculating the absolute and relative errors in the approximate values of $y(1)$ and $y(2)$ for both $n = 2$ and $n = 4$. **Please show at least 4 digits after the decimal point.** If you want to do the error calculations in MATLAB, note that the MATLAB syntax for the absolute value function is `abs`

$$\text{Absolute Error} = |y_{\text{exact}} - y_{\text{approx}}| \qquad\qquad \text{Relative Error} = \left| \frac{y_{\text{exact}} - y_{\text{approx}}}{y_{\text{exact}}} \right|$$

	exact solution	R-K Method, $n = 2$	R-K Method, $n = 4$
$y(1)$	A	C	E
Absolute Error at $x = 1$	0		
Relative Error at $x = 1$	0		
$y(2)$	B	D	F
Absolute Error at $x = 2$	0		
Relative Error at $x = 2$	0		