The Isochron Method for Determining the Age of a Rock

Background Information

Knowledge of radioactive decay processes provides one way to estimate the age of rocks and fossils containing a radioactive isotope. The basic idea is to measure the amount of radioactive substance now in the rock and to subtract that quantity from the amount originally present, giving the amount of substance that has decayed since the rock was formed. Knowing how long it would take this amount of substance to decay, one can deduce the age of the rock. One problem with this method is that we need to know how much of the substance was originally present. The isochron method avoids this problem.

Certain rocks contain more than one type of mineral. Consider a rock containing five different minerals, each of which contains isotopes of the elements rubidium (Rb) and strontium (Sr). Rubidium 87 (^{87}Rb) is radioactive with a half-life of 47 billion years, decaying directly to strontium 87 (^{87}Sr). Three other isotopes of strontium are also present in the rock: ^{88}Sr, ^{86}Sr, and ^{84}Sr; none of these isotopes is produced by radioactive decay.

Suppose we now represent each of the five minerals present in the rock by a data point on a graph. The first coordinate of each data point is the ratio of the original amount of ^{87}Rb to the original amount of ^{86}Sr; the second coordinate is the ratio of the original amount of ^{87}Sr to the original amount of ^{86}Sr.

Since the isotopes of strontium are chemically identical, the ratio of ^{87}Sr to ^{86}Sr will be the same for each mineral when the rock is formed. (The ratio ^{87}Sr /^{86}Sr in each mineral just equals the value of this ratio in the region where the rock was formed.) Therefore, the five data points lie on a horizontal line at time 0, the time at which the rock was formed.

As time goes on, the amount of ^{87}Rb in each mineral decreases because of radioactive decay, and the amount of ^{87}Sr increases. (The amount of ^{86}Sr remains constant.) The more ^{87}Rb a mineral starts out with, the more ^{87}Sr it will contain at later times. Therefore, if we plot ^{87}Sr / ^{86}Sr vs. ^{87}Rb / ^{86}Sr for each mineral at time $t > 0$, the points will no longer lie on a horizontal line. It turns out that these points lie on a line with positive slope. The slope is related to the amount of time t that has elapsed since the rock was formed.

Your tasks are to explain why data points must lie on a straight line even for $t > 0$ and to compute the slope of this line in terms of t and the half-life of ^{87}Rb.

Problem Formulation

Let t denote time (in billions of years); let r_1 denote the amount (in moles) of ^{87}Rb in mineral 1 at time t; let s_1 denote the amount (in moles) of ^{87}Sr in mineral 1 at time t; let u_1 denote the amount (in moles) of ^{86}Sr in mineral 1 at time t; let r_2 denote the amount (in moles) of ^{87}Rb in mineral 2 at time t; etc. Let τ denote the half-life of ^{87}Rb.

1. Find a formula for r_1 in terms of t, τ, and r_{10}, the amount of ^{87}Rb in mineral 1 at time 0. Do the same for r_2, r_3, r_4, and r_5.
2. Find a formula for s_1 in terms of t, τ, s_{10}, and r_{10}. Do the same for s_2, s_3, s_4, and s_5.
3. Show that the points $(r_1/u_1, s_1/u_1)$, $(r_2/u_2, s_2/u_2)$, … $(r_5/u_5, s_5/u_5)$ lie on a straight line, and find the slope of this line in terms of t and τ.

Reference

Miller, Kenneth R. (1984). "Scientific Creationism Versus Evolution: The Mislabled Debate," in *Science and Creationism*, Ashley Montagu, ed., Oxford University Press, 18 - 63.