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Dynamics of Offshore Wind Turbines

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ABSTRACT
The dynamic behavior of offshore wind turbines decides the design of several components, such as bearings, gear boxes, foundation platform and tower. In this paper, an analytical rotordynamic model of offshore wind turbine is developed and effects of several parameters, such as rotor blade size, rotational speed, distribution of mass imbalance, wind loading, wave loading, current speeds, and location of rotor-nacelle assembly are investigated on the dynamic behavior (response and stability) of an offshore wind turbine. It is particularly important to determine the rotor response for minimizing structural resonance and corresponding failure of wind turbine components. The dynamic response prediction is also desirable to address the fatigue design of wind turbine components. The rotordynamic model of the wind turbines is expressed as partial differential equations, with time-dependent boundary conditions. These equations of motion are integrated with respect to time for a simple case of rotor imbalance to determine the closed-form solutions of the rotor response and its resonance frequencies, in the absence of wind or wave loadings. The effect of wave and wind loadings is described in the governing equations of motion. The predictions developed through the analytical model for the wind turbines free vibrations are compared with finite element modeling of the wind turbines. The results are compared and the relative merit of using either analytical or finite element modeling is discussed.

KEY WORDS: Wind turbines; dynamics; vibrations; rotordynamics; tower; rotor-nacelle.

NOMENCLATURE

ρ	Mass density of the tower
D	Tower outside diameter
A	Cross-sectional area of the tower
E	Young's Modulus of the tower
I	Second moment of area
l	Height of the tower
h	Water depth
y	Vertical coordinate measured from tower base
η	Transverse displacement coordinate
t	Time
M	Mass of the nacelle
m	Combined mass of the rotor blades
Ω	Rotational speed of the rotor
ω	Eigenvalue of the rotor-nacelle-tower assembly in units of Radians per Second
C_q	Tower characteristic number
f_i	Eigenvalue of the assembly in units of Hertz
e	Rotor blades effective eccentricity

INTRODUCTION
Offshore wind turbines are an important part of quest for energy resources, as the renewable energy takes on added value in view of depleting resources from onshore and offshore locations around the world. Offshore wind turbines are a relatively new concept in renewable energy. The technical challenge for their design and analysis stems from the effect of high winds, wave and current loadings. The main reason for the rapid growth and development of offshore wind turbines is relative abundance of winds in the offshore regions, social and cultural issues related to site obstructions, and noise levels which can be tolerated offshore as compared to onshore.

The offshore wind turbines are currently being designed and constructed for the following configurations: (1) Monopile; (2) Tripod; and (3) Jacket. These terms refer to the type of supporting offshore platform structure in a particular environment.

In the past two decades, finite element analysis (FEA) has gained immense popularity due to its power to model complex engineering problems, and developing powerful solutions to engineering designs all around the world, which is not just limited to oil, gas and renewable energy industry. Commercially, several different leading finite element softwares are available, with advanced user interface and fully built-in nonlinear, three-dimensional capabilities which are utilized to solve sophisticated engineering problems.

In the design process of offshore and onshore wind turbines, several finite element softwares are used, either independently, or sometimes in conjunction with each other to provide design solutions to the manufacturers and designers all around the world. Several leading softwares include FAST, ADAMS, AeroDYN, Abaqus Standard, ANSYS, ALGOR and many other softwares.

While finite element softwares are indispensable tools for the detailed solution of all the engineering problems in today's world, an engineer still needs to be familiar with the basic theory and standard analytical models to have a better judgment of the outputs from a software program. This paper provides a basic dynamic analytical model of a wind turbine with the governing partial differential equations of motion and their solution for a simple case. The structure of the governing equations of motion will show the engineer the effect of different physical parameters on the response and characteristics of a wind turbine.

The model is developed for transverse vibrations of a wind turbine, and considers the effect of rotor imbalance only. The gyroscopic effects are not considered.

The framework of governing equations of motion presented in this paper highlights how the wave, wind and current loading can be incorporated to predict the time response of a wind turbine under offshore conditions.

LITERATURE REVIEW

Onshore and offshore wind turbine design and analysis is performed in the industry mostly using advanced finite element based softwares. The mechanical modeling of wind turbines in the form of governing equations of motion (rotordynamic analysis) is not available in the public domain. The only information available in this connection is the analysis performed using finite element software packages on specific projects or hypothetical cases. The generator modeling (electrical aspects such as power, voltage and current) is available in public domain. Correspondingly, there is generally a lack of physical understanding of physical principles and parameter effects on wind turbine dynamic response.

Lavassas et.al. (2003) outline the modeling and simulation of a 1 MW offshore wind turbine using advanced finite element modeling tools. The tower is 44,000 m in height, with a variable cross-section tower. This is a three-bladed cantilevered wind turbine. The structural response of the wind turbine is investigated for the design of the wind turbine. The wind turbine is analyzed for gravity, seismic and wind loading. It should be noticed that this is an example of an onshore wind turbine, but illustrates the design approach.

Long and Moe describe the modeling and analysis of two different wind turbine configurations in offshore conditions. The model is simulated with a jacket structure supporting the tower, rotor and nacelle. The advanced finite element modeling tools are used for full three-dimensional modeling of the entire assembly and to simulate the wave and wind loadings on the structure.

MATHEMATICAL MODEL

(a) Equation of Motion

The model of a wind turbine tower with the coordinate systems used to describe the dynamics is shown in Fig. 1:

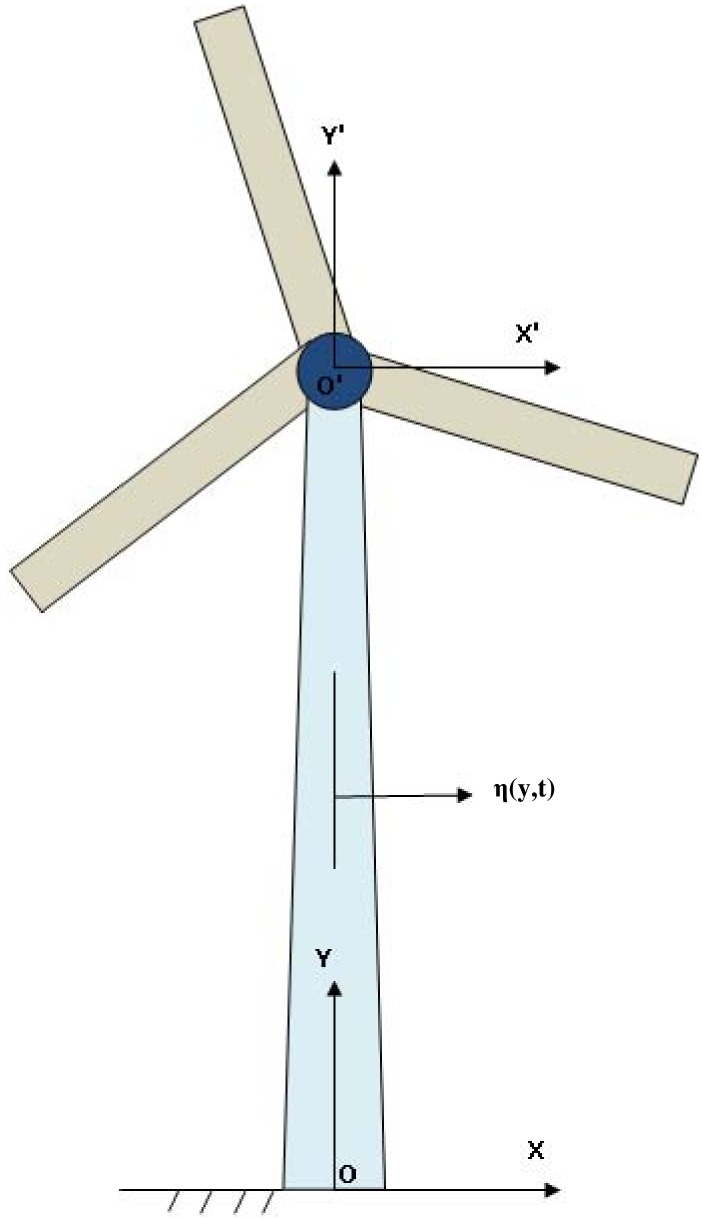

Fig. 1: Wind turbine model

The coordinate system fixed to the ground at the tower base is OXY, whereas another fixed coordinate system (O'X'Y') is fixed at the static position of the nacelle. Three blades are shown for representative purposes only.

The governing partial differential equation of motion for the transverse vibration of the tower is expressed as (Williams, 1996):

$$\rho A \frac{\partial^2 \eta}{\partial t^2} = -\frac{\partial^2}{\partial y^2}\left(EI \frac{\partial^2 \eta}{\partial y^2}\right) + f(y,t) \tag{1}$$

The geometric boundary conditions of the model are as follows:

$$\eta(0,t) = 0 \quad (2)$$

$$\frac{\partial\eta(0,t)}{\partial y} = 0 \quad (3)$$

These are the conditions imposed assuming the fixed base of the tower.

The natural boundary conditions of the model are expressed as follows:

$$\frac{\partial}{\partial y}\left(EI\frac{\partial\eta}{\partial y}\right) = 0 \quad (4)$$

$$(M+m)\frac{\partial^2\eta}{\partial t^2} = \frac{\partial}{\partial y}\left(EI\frac{\partial^2\eta}{\partial y^2}\right) + me\Omega^2\cos(\Omega t) \quad (5)$$

The natural boundary conditions are applicable at y=l (location of tower nacelle).

The effect of imbalance is to impose a time-dependent boundary condition at the top of the tower, through harmonic excitation.

(b) Eigenvalue Characteristic Equation

The characteristic equation for the tower-nacelle-rotor assembly is given by the following equation:

$$-(M+m)\beta\left(\frac{EI}{\rho A}\right)(sin\beta l cosh\beta l - cos\beta l sinh\beta l) + EI(1 + cos\beta l cosh\beta l) = 0 \quad (6)$$

This characteristic equation applies for β_i, where i=1,2,3,…..The roots of this nonlinear equation yield the values of β_i, which leads to the eigenvalues of the system, for given system parameters, such as mass of rotor and nacelle, tower cross-section and height.

The parameter 'β' is defined from the following equation:

$$\omega_i = \beta_i^{\,2}\sqrt{\frac{EI}{\rho A}} \quad (7)$$

The eigenvalues ω_i are the mode frequencies of the system expressed in units of radians per seconds. From ω_i, the mode frequencies can be calculated in the units of cycles per second (Hz) as follows:

$$f_i = \frac{\omega_i}{2\pi} \quad (8)$$

(c) Mode Shapes

The mode shape coefficients ratios for various modes are calculated using the following equation:

$$\frac{c_n}{d_n} = -\frac{sin\beta_n l + sinh\beta_n l}{cos\beta_n l + cosh\beta_n l} \quad (9)$$

The mode shapes for the rotor-tower-nacelle assembly are calculated from the following equation:

$$Y_n(y) = d_n\left[\frac{c_n}{d_n}(cos\beta_n y - cosh\beta_n y) + (sin\beta_n y - sinh\beta_n y)\right] \quad (10)$$

(d) Tower Response

The rotor-tower-nacelle assembly space-time response solution is given by the following equation:

$$\eta(y,t) = \frac{me\Omega^2}{\Delta}\cos(\Omega t)\,[(sin\beta_n l + sinh\beta_n l)(\cos\beta_n y - cosh\beta_n y) - (cos\beta_n l + cosh\beta_n l)(sin\beta_n y - sinh\beta_n y)] \quad (11)$$

The denominator of the response function in the previous equation is defined as:

$$\Delta = -2\left((M+m)\Omega^2\right)(cosBlsinhBl - sinBlcoshBl) - 2EIB^3(1 + cosBlcoshBl) \quad (12)$$

$$B = \sqrt{\frac{\Omega}{C_q}} \quad (13)$$

$$C_q = \sqrt{\frac{EI}{\rho A}} \quad (14)$$

This equation is the same as the characteristic equation, except that the eigenvalues are replaced by the rotor speed, which causes rotating imbalance vibrations to occur at the same frequency as the rotor speed (synchronous vibration). Therefore, if the rotor speed coincides with any eigenvalues, the denominator becomes zero, and the resonance occurs.

Based on above analysis, the wind turbine designer is most concerned with designing the rotor-nacelle-tower assembly in such a way that the rotor speed is significantly different from the first eigenvalue of the assembly. In most cases, the first mode is the critical mode, because it typically ranges between 15 cpm to 30 cpm, which is the speed range in which most commercial wind turbines operate.

The rotordynamic boundary condition is derived by analyzing the rotor-nacelle mass in the tower top fixed coordinate system. This is rotating imbalance condition, which is derived in detail for dynamics of continuous systems (Williams, 1996 and Thomson, 1992).

(e) Effect of Wave and Winds

The effect of waves and winds can be accounted for by introducing the resulting force due to wave and wind by a space-time dependent function in the equation of motion as follows:

$$\rho A\frac{\partial^2\eta}{\partial t^2} = -\frac{\partial^2}{\partial y^2}\left(EI\frac{\partial^2\eta}{\partial y^2}\right) + f(y,t) \quad (1)$$

The wave and wind resultant forces can be modeled as one dimensional force, even though in practice they are function of more than merely the tower height coordinate, y. Nevertheless, this approximation is still useful to obtain some analytical solutions, if sufficiently simplified models of the wave and wind loading are introduced.

The procedure for obtaining the eigenvalues and eigenvectors is the same. However, the response will now be a superposition of the effects of the rotating imbalance and the distributed forcing function, f(y,t). For offshore wind turbines, the forcing function f(y,t) in general, will be a piecewise function, since the wind acts over a certain height at which the wave loads do not act. Similarly, the waves act below a certain height, at which the winds do not act.

For modeling an onshore wind turbine, the force f(y,t) will represent only the wind loading effects, and not the wave loads, since the structure is not installed under water and hence there are external loadings due to wind only.
The complete response solution will be a superposition of the piecewise forcing function and the rotating imbalance.
As an example, the wind loading can be modeled as constant across the tower height, or it can also be modeled as varying linearly across the tower height.
In the former case, the function f(y,t) is simply:

$$f(y,t) = C$$

In the later case (linear variation), the function f(y,t) is given as:

$$f(y,t) = \frac{C(y-h)}{l-h}$$

Note that this assumes that the wind velocity is steady and independent of time. However, the wind velocity can also be assumed as a function of time, in which case:

$$f(y,t) = Cg(t)$$

$$f(y,t) = \frac{C(y-h)}{l-h}g(t)$$

This part of the governing differential equation can be solved in the same way as the first part, namely, the separation of variables.

For the case of constant, steady wind profile, the solution is simply:

$$\eta(y,t) = C'(t^2 + t + C_0)$$

For the case of linearly varying, steady wind profile, the solution is simply:

$$\eta(y,t) = C'(y-h)(t^2 + t + C_o)$$

This solution is added to the other part of particular solution (equation (11)) to obtain the complete solution.

RESULTS OF MATHEMATICAL MODEL

(a) Model Parameters

The model parameters for the wind turbine model are described as follows:
Young's Modulus = E = 30 MPsi (typical steel grade)
Tower outside diameter = 15 ft.
Tower thickness = 3.75 in.
Tower height = 300 ft.
Nacelle mass = M= 200,000 kg
Rotor blades mass (total blade mass) = m = 50000 kg
Rotor imbalance (eccentricity) = e = 2 in.
Rotational speed = Ω = 20 revolutions per minute (rpm)

The first eigenvalue is 0.34 Hz (22.4 cpm), the second eigenvalue is 2.72 Hz (162.5 cpm), while the third eigenvalue is 8.1 Hz (483 cpm). Of particular importance is the first eigenvalue, which typically lies in the operating speed range of most of the commercial wind turbines (about 15 cpm-30 cpm). This necessitates introducing damping in the rotor-nacelle assembly and stiffening the structure through design modifications to ensure that the first eigenvalue is out of the speed range in order to avoid resonance of the wind turbine.

With these parameters, the solution of the governing equations of the mathematical model is described by the following graphs.

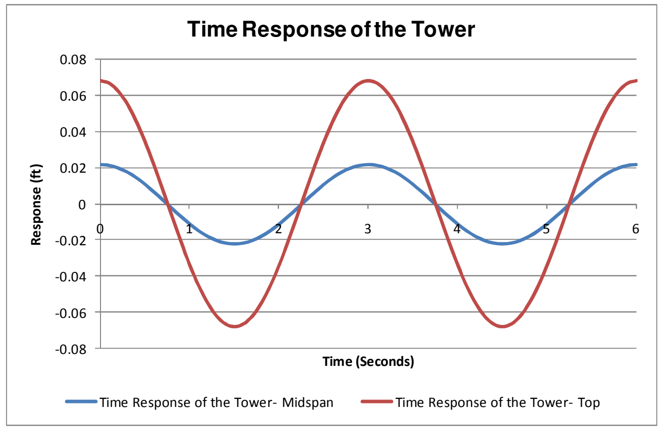

Fig. 2: Time Response at middle and top of tower

Fig. 2 shows the time response at the top (nacelle) and mid-span tower locations. The solutions show that the time responses are periodic, harmonic, with the period equal to the inverse of the rotational speed (in rad/s). This response is purely due to rotational imbalance in the system. The solution also shows that the top tower response is several times higher than the mid-span response, and that both locations show the responses to be in –phase with each other.

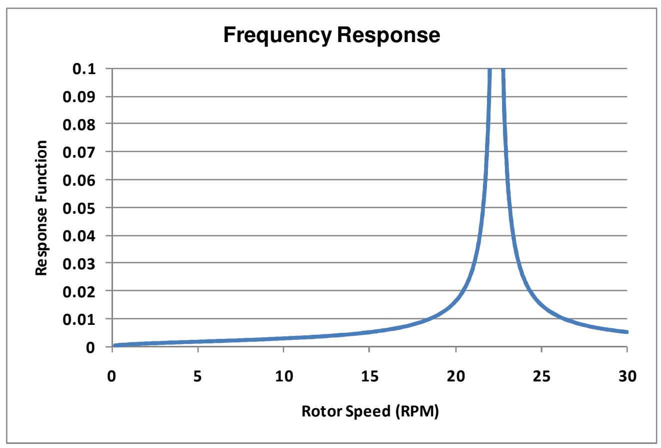

Fig. 3: Amplitude response of the tower as a function of rotational speed

Fig. 3 shows the frequency response of the amplitude of wind turbine vibrations at the tower nacelle location. This is a plot of equation (11) (the amplitude term).The diagram shows a typical resonance location, which for the given parameters chosen is about 22.4 rpm. This shows the significance of damping mechanisms (not included in the model) to minimize and control the vibration amplitudes near the resonance, and selecting an operating speed significantly away from the resonance.

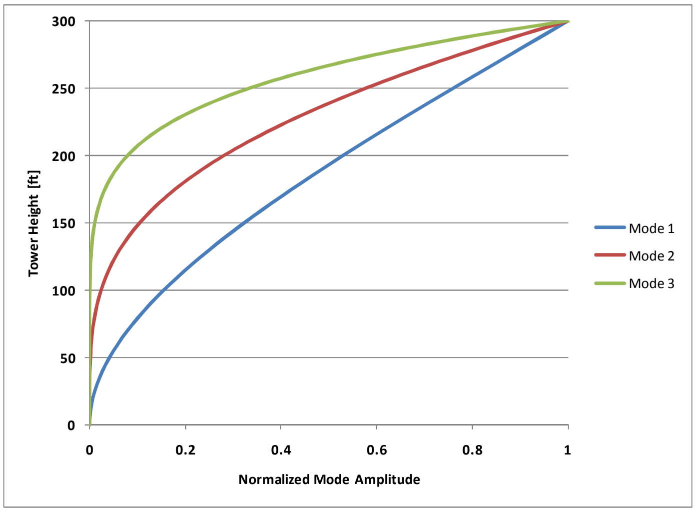

Fig. 4: Tower mode shapes

Fig. 4 shows the first three modes of vibrations of the tower. This is the plot of equation (10) for the three modes.

(b) Effect of varying model parameters

The effects of model variation can be easily predicted using the analytical model. The following parameter sensitivity cases are considered:

1. Rotor-nacelle mass
2. Tower height
3. Tower diameter

The sensitivity cases are considered such that only one parameter is varied while the others are considered fixed. The effects of these parameters can be described as follows:

The rotor-nacelle mass corresponds to the inertia of the wind turbine. The higher the inertia is the smaller is the natural frequency of the system and vice versa. The tower height and tower diameter correspond to the flexibility of the wind turbine. The higher the tower height is, the more flexible the tower is and hence the lower the natural frequencies will be. On the other hand, the tower diameter corresponds with the stiffness of the wind turbine. With larger diameter, more stiff the wind turbine will be, and hence the natural frequencies will be higher. In this context, the tower height and diameter can be considered to contribute oppositely to the overall stiffness of the wind turbine, and hence to the natural frequencies of the system.

The numerical results validate the above conclusions. For illustration, with a fixed tower diameter of 15 ft and combined rotor-nacelle mass of 250,000 kg (550 kips), the sensitivity of the first three modes of the wind turbine tower to the tower height are shown in Table 1:

Table 1. Effect of varying tower height on first three modes of the wind turbine (mode values in Hz)

Parameter: Tower height	Tower diameter = 15 ft (4.6 m)/ Rotor-Nacelle Mass = 550 Kips(250,000 kg)		
	Mode 1	Mode 2	Mode 3
300 ft	0.34	2.72	8.1
350 ft	0.28	2.02	5.95
400 ft	0.22	1.56	4.6

Table 2. Effect of varying rotor-nacelle mass on first three modes of the wind turbine (mode values in Hz)

Parameter : Rotor-Nacelle Mass	Tower height = 300 ft (91.5 m)/ Tower diameter = 15 ft		
	Mode 1	Mode 2	Mode 3
200,000 kg	0.39	2.76	8.1
250,000 kg	0.37	2.72	8.1
300,000 kg	0.35	2.66	7.99

The results of the analytical model from Table 1 and Table 2 show that the natural modes change significantly with tower height, and that the modes are slightly less sensitive to the rotor-nacelle mass. Nevertheless, the variation in rotor-nacelle mass still affects the eigenvalues of the wind turbine. This parameter sensitivity knowledge o structural eigenvalues is important for a design engineer, to select an appropriate combination of parameters in order that the design rotational speed of the rotor can be significantly away from the wind turbine first eigenvalue to avoid structural resonance due to rotating imbalance.

(c) Effect of variable cross-sectional area

The effect of variable cross-sectional area can be incorporated into the governing differential equation (1), which affects both the cross-sectional and the second moment of area. For commercial wind turbines, the tower typically has a tapered cross-sectional area, which can be several joints. It can be simplified as a tapered truncated cone, and the area can be expressed as a function of the tower height, y.

As illustration, the tower diameter can be expressed as a function of height coordinate 'y' as:

$$D(y) = D_{01}\left(1 - \frac{y}{l}\right) + D_{02} \tag{15}$$

In this equation, the quantities D_{01} and D_{02} are the diameter of the tower at top and base, respectively.

An analytical solution can still be obtained for simple enough variation as shown in equation (15).

FINITE ELEMENT MODEL

A simple finite element model for the wind turbine assembly is developed using Abaqus/CAE™. The elements are three-dimensional beam elements with two nodes. The model is shown in Fig. 5.

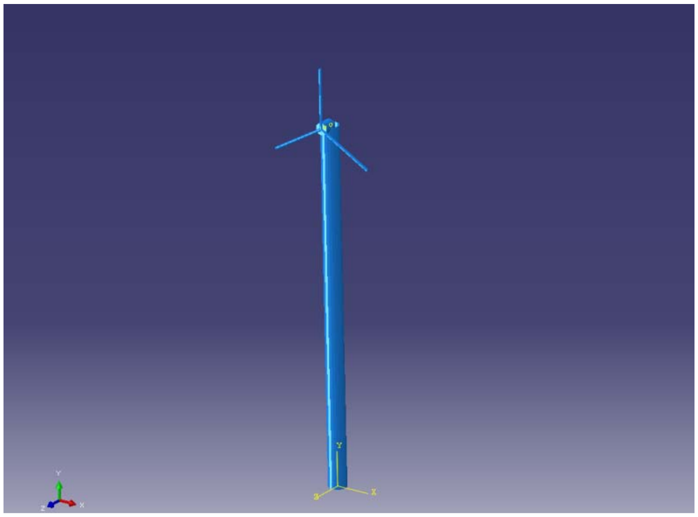

Fig. 5: Finite element model of the wind turbine (rendered beam profile image from Abaqus/CAE™)

The finite element model has the same parameters as the analytical model parameters presented in the preceding section. The model shows high agreement with the analytical model for the eigenvalues and eigenvectors.

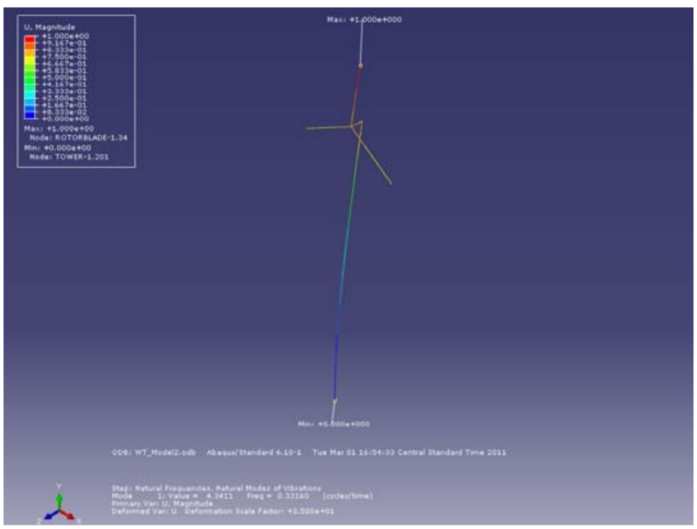

Fig. 6: Wind turbine model's first mode of vibration

As an illustration, Fig. 6 and Fig. 7 show the wind turbine views in the first mode, which are that of a cantilevered beam, with base fixed (geometric boundary condition) and rotor-nacelle interface at highest amplitude.

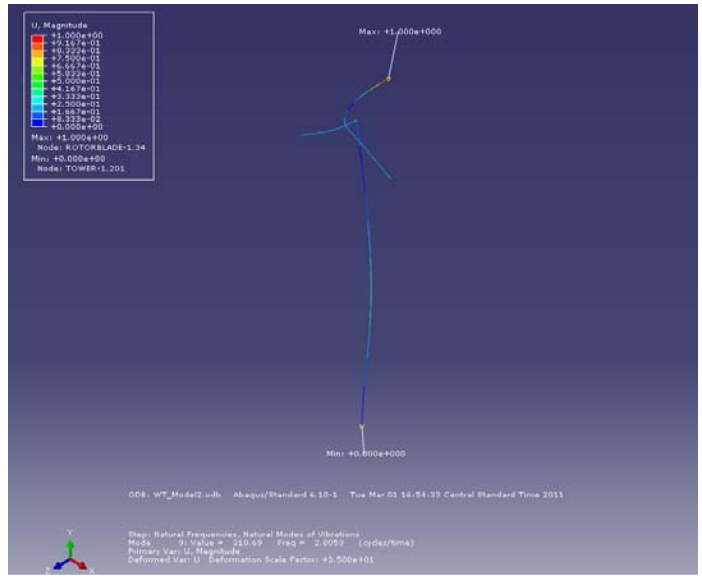

Fig. 7: Wind turbine model's fourth mode of vibration (flexing blade and tower)

COMPARISON OF ANALYTICAL AND FINITE ELEMENT MODELS

The comparison of the analytical model with the finite element model represents good agreement between the eigenvalues and the eigenvectors, thereby showing that the first mode of real significance for wind turbines is the transverse vibration mode.

The comparison of the tower natural frequencies (in which tower participates significantly) from FEA and analytical method is presented in the table below:

Table 3: Comparison of FEA and analytical formulations

Natural Frequencies (Hz)	**Method of Analysis**	
	FEA	Analytical
First Mode	0.332	0.34
Second Mode	2.805	2.72
Third Mode	8.342	8.10

It should be noted that the FEA model shows several blade vibration modes between 2.72 Hz and 8.1 Hz, which are not modeled in the analytical method. The tower predominantly participates in the modes which are 0.34 Hz, 2.72 Hz and 8.1 Hz, and for higher modes.

CONCLUSIONS

The conclusion from the basic work presented in this paper is that the governing differential equations for the transverse vibrations of the wind turbines are solvable and tractable for simple boundary conditions. The structure of the partial differential equations with different boundary conditions together with the effect of rotating imbalance is shown, and the importance of different parameters on the response of the system is emphasized. The comparison with a three-dimensional wind turbine model is presented, and it is shown that the transverse vibrations, as compared to axial and torsional vibrations, are of real significance to the design of a wind turbine. The comparison also shows good agreement between the analytical solution and the finite element solution. The presented work encourages the design engineer to use the presented equations and their solutions as a preliminary check and insight into the dynamics of the system before engaging in advanced and detailed modeling of the wind turbine using rotordynamic and finite element softwares.

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