92.236 Engineering Differential Equations
Example of Solution Procedure for a First Order Linear DE

Solve the d.e. $xy' = 3y + 4x$. (This is a linear d.e. because y and y' appear only to the first power, multiplied only by functions of x.)

Step 1. Write the d.e. in the standard form $y' + P(x)y = Q(x)$:

$$xy' = 3y + 4x \Rightarrow xy' - 3y = 4x \Rightarrow y' - 3y/x = 4x/x \Rightarrow y' - \frac{3}{x}y = 4. \tag{1}$$

Step 2. Find the integrating factor:

$$\rho(x) = e^{\int P(x)\ dx} = e^{\int -\frac{3}{x}\ dx} = e^{-3\ln(x)} = \left[e^{\ln(x)}\right]^{-3} = x^{-3}. \tag{2}$$

Notice that the minus sign is part of $P(x)$.

Step 3. Multiply the standard form of the d.e. by the integrating factor:

$$x^{-3}\left[y' - \frac{3}{x}y\right] = x^{-3}\,[4] \Rightarrow x^{-3}y' - 3x^{-4}y = 4x^{-3}. \tag{3}$$

Step 4. Use the Product Rule backwards to write the d.e. in the form $\frac{d}{dx}\left[\rho(x)y\right] = \rho(x)Q(x)$:

$$\frac{d}{dx}\left[x^{-3}y\right] = 4x^{-3} \tag{4}$$

It's a good idea to apply the Product Rule to the left side of equation (4) to make sure equations (4) and (3) are equivalent.

Step 5. Integrate both sides:

$$x^{-3}y = \int 4x^{-3}\ dx = -2x^{-2} + c. \tag{5}$$

Step 6. Solve for y: $y = \dfrac{-2x^{-2} + c}{x^{-3}} \Rightarrow \boxed{y = -2x + cx^3}$