MATH.2360 Engineering Differential Equations – Phase Planes

**Due Date: Friday, April 5**

A *phase plane* is the two-dimensional analog of a *phase line* for a first-order autonomous d.e.

The phase line for the autonomous d.e. $dx/dt = f(x)$ is a copy of the $x$ axis with arrows indicating the direction in which the solution $x(t)$ moves (in the $+x$ direction or in the $-x$ direction).

The phase plane for the second-order autonomous d.e. $x'' = f\left(x, x'\right)$ is a copy of the $x$-$x'$ plane with arrows indicating the direction in which the "orbit" $(x(t), x'(t))$ moves. Since it is difficult to determine the behavior of solutions in two dimensions just by looking at direction arrows, we usually draw a few orbits in addition to drawing the direction arrows.

For example, consider the equation $x'' = -\sin(x)$, which models the motion of an undamped pendulum. (The variable $x$ represents the angle the pendulum makes with a vertical line through the pendulum's pivot, and the primes denote derivatives with respect to time $t$.) The orbits are curves in the $x$-$x'$ plane whose coordinates are the position and angular velocity of the pendulum at different times.

**Here is what I am asking you to do:**

- Download the file phaseplane.m from our course web page. (Click on the *Class handouts etc.* link under the **Course Materials** heading, then look under *MATLAB Handouts*.)
- Start MATLAB and change to the directory in which you saved phaseplane.
- Before using phaseplane, create a file defining the d.e. whose phase plane you want to see. For this assignment, the d.e. is $x'' = -\sin(x)$.
  Click on *New* on the toolbar, then click *Script*. An Editor/Debugger box will open on the screen. Type in the following five lines (including the last line with the word end):
  ```
  function w = f(t, z)
  x = z(1);
  y = z(2);
  w = [y; -sin(x)];    % Note the ; between the 2 components of w
  end
  ```
  (Don't forget the semicolon at the end of the second, third, and fourth lines.) Save the function file using the name f.m
- In the MATLAB Command window, type the command
  `phaseplane(@f,-6,6,-4,4)`
- In response to the prompt in the command window, enter the point `[0, 1]` and hit *Enter*.
- In response to the prompt in the command window, enter the point `[0, 3]` and hit *Enter*.
- In response to the prompt in the command window, just hit *Enter*.
- Click on *Edit* on the Figure window toolbar, then click on *Copy Figure*. Open a Word document and paste in the figure you just copied.

**Five bonus points will be added to your homework point total if you turn in the phase plane display. Five additional bonus points will be added to your homework point total if you explain the physical meaning of the two orbits in terms of how the pendulum is moving. How does the pendulum motion starting at $(0, 1)$ differ from the motion starting at $(0, 3)$?**

Please email your document to me at `stephen_pennell@uml.edu`