92.236 Engineering Differential Equations – John Polking's Phase Plane Tool *pplane8*

Due Date: Friday, March 27

A *phase plane* is the two-dimensional analog of a *phase line* for a first-order autonomous d.e.

The phase line for the autonomous d.e. $dx/dt = f(x)$ is a copy of the x axis with arrows indicating the direction in which the solution $x(t)$ moves (in the $+x$ direction or in the $-x$ direction).

The phase plane for the second-order autonomous d.e. $x'' = f\left(x, x'\right)$ is a copy of the x-x' plane with arrows indicating the direction in which the "orbit" $(x(t), x'(t))$ moves. Since it is difficult to determine the behavior of solutions in two dimensions just by looking at direction arrows, we usually draw a few orbits in addition to drawing the direction arrows.

For example, consider the equation $x'' = -\sin(x) - Dx'$, which models the motion of a damped pendulum. (The variable x represents the angle the pendulum makes with a vertical line through the pendulum's pivot, and the primes denote derivatives with respect to time t.) The orbits are curves in the x-x' plane whose coordinates are the position and angular velocity of the pendulum at different times.

Here is what I am asking you to do:

- Download the file pplane8.m from our course web page. (Click on the *Class handouts etc.* link under the **Course Materials** heading, then look under *MATLAB Handouts*.)
- Start MATLAB and change to the directory in which you saved pplane8.
- Type the command

  pplane8

  in the MATLAB command window.
- In the pplane8 Setup box, click on *Gallery* on the toolbar, then click on *pendulum*. (Polking uses the letter θ instead of x, and he uses ω to denote x'.)
- Set the minimum and maximum values of θ to be -3 and 3, respectively, and set the minimum and maximum values of ω to be -3 and 3, respectively.
- Click on the *Proceed* button in the bottom right of the pplane8 Setup box.
- In the pplane8 Display box, click on *Options* on the toolbar, then *Solution direction*, then *Forward*.
- In the pplane8 Display box, click near the point $(1, 0)$ to generate an orbit.
- Click on *Edit* on the toolbar, then click on *Copy Figure*. Open a Word document and paste in the figure you just copied.
- Now go back to the pplane8 Setup box, change the value of the damping coefficient D from 0 to 0.1, then click on the *Proceed* button.

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- In the pplane8 Display box, click near the point $(1, 0)$.
- Copy the figure to your Word document.
- When you are finished, click on the Quit button in the pplane8 Display box, then close MATLAB.

Five bonus points will be added to your homework point total if you turn in the two phase plane displays. Five additional bonus points will be added to your homework point total if you explain the physical meaning of the two orbits in terms of how the pendulum is moving. How does the pendulum motion with $D = 0$ differ from the motion with $D = 0.1$?

Please email your document to me at `stephen_pennell@uml.edu`