Engineering Differential Equations

Some Useful MATLAB Commands for Symbol Manipulation

A. Algebra

1. MATLAB will factor polynomials. Try the commands
   syms x % This tells MATLAB to treat x as a symbol rather than an array of numbers
   factor(x^4 - 5*x^2 + 4)

2. MATLAB can solve single equations. Try
   solve('x^4 - 5*x^2 + 4 = 0')

3. MATLAB can also solve systems of equations:
   [x, y] = solve('2*x + y = 5', 'x + 2*y = 4')

4. MATLAB can simplify expressions.
   Try these commands to simplify $\dfrac{z}{2z+1} + \dfrac{1}{z}$ and $\cos^3(x) + \cos(x)\sin^2(x)$
   clear x y
   syms x z
   simplify (z / (2*z + 1) + 1 / z)
   pretty(ans) % The "pretty" command makes the output easier to read.
   simplify ((cos(x))^3 + cos(x) * (sin(x))^2)

B. Calculus

MATLAB can find limits, derivatives, and integrals symbolically. Try the following commands to find $\lim_{x\to 0} \dfrac{\sin(x)}{x}$, $\lim_{x\to\infty} e^{-x}$, $\dfrac{d}{dx}\left[x^3\right]$, $\dfrac{d^2}{dx^2}\left[x^3\right]$, $\displaystyle\int \frac{1}{x^2+1}\,dx$, and $\displaystyle\int \frac{x}{(x^2+1)^{3/2}}\,dx$, respectively.

limit(sin(x)/x, x, 0)

limit(exp(-x), x, inf) % inf means infinity

diff(x^3) % The diff command takes a derivative

diff(x^3, 2) % The 2 means take the second derivative

int(1 / (x^2+1)) % The int command integrates

pretty(int(x / (x^2+1)^(3/2)))

OVER

C. Differential Equations

MATLAB can even solve differential equations symbolically. Here are some examples:

1. To solve the d.e. $x^2y' + 2xy = 3x^2$, type the command

   dsolve('x^2 * Dy + 2*x*y = 3*x^2', ' x ') % Dy denotes the derivative dy/dx.

   %The x in the command tells MATLAB that x is the independent variable

2. You can also solve initial value problems, such as $y' = y(1 - y),\ y(0) = 1/2$:

   dsolve('Dy = y*(1 - y), y(0)=1/2', ' x ')

3. Second derivatives are denoted D2y, third derivatives D3y, etc. To solve the second-order equation $y'' + y = 0$, type

   dsolve('D2y + y = 0', ' x ')

4. Of course, you can also add initial conditions to this problem, and you can give the solution a name. You can even calculate the value of the solution at any x value you like using the **subs** command.

   y = dsolve('D2y + y = 0, y(0)=1, Dy(0)=1' , ' x ')

   subs(y, pi/4)

D. Exercises (These are from the homework for section 1.5.)

1. Solve the initial value problem $y' + y = 2,\ y(0) = 0$.
2. Solve the initial value problem $xy' - y = x,\ y(1) = 7$.