DUE DATES: FRIDAY, APRIL 3.
OPTIONAL REVISION DUE FRIDAY, APRIL 24.

No extensions will be given except in case of illness or other emergency.

General Guidelines

1. You may work in groups of up to four. Turn in just one paper for the whole group. Each member of the group will receive the same grade. **If you would like to work in a group but cannot find a group to join, please let me know no later than Friday, March 13.**

2. If you choose to turn in a revised version of your work, 90% of your grade will be based on the final version and 10% on the first version.

3. This is a major assignment, and you should plan to do some work on it every day, starting today. **DO NOT WAIT UNTIL THE LAST MINUTE. If you do, two things will happen: you will not be able to finish the project, and I will have no sympathy for you.**

4. The report you turn in must be your group's work. I reserve the right to ask any or all of you for a verbal explanation of your solution.

5. I strongly encourage you to see me regularly as you work on the project to discuss your progress. If you receive any assistance from anyone other than me, of if you use our textbook or any other outside source, you must cite the source in your paper. Remember guideline number 4.

What Should You Hand In?

You should fill in the blanks on the attached answer sheet, and you should attach additional pages showing how you arrived at these answers and showing how you derived equation (11).

Background Information

The purpose of this project is to investigate transverse vibrations of wind turbine towers and to look for possible resonance. The figure below illustrates a wind turbine tower. Given the proper initial conditions, such a tower can oscillate in the horizontal direction. You will first analyze this oscillatory motion in the absence of damping and external forcing, and you will then determine whether forcing from the rotation of the turbine blades can lead to resonance (which could be bad news for the turbine tower).

Jafri et al. describe a mathematical model of transverse vibrations of a wind turbine tower using variables with physical dimensions. In order to reduce the number of parameters in the problem, we will work with a model using scaled, dimensionless variables.

Wind Turbine Tower
(after Jafri et al.)
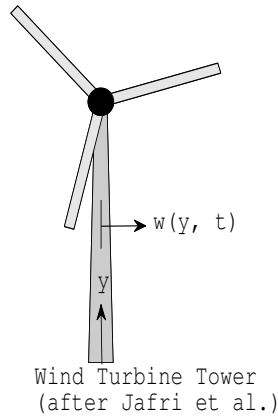
In the remainder of this project description, the following notation will be used.

t	time (dimensionless)
y	height above ground level (dimensionless)
$w(y,t)$	horizontal displacement of tower center line (dimensionless)

It can be shown that in the absence of damping and external forcing, $w(y,t)$ (the horizontal displacement of the tower center line) is described by the following differential equation and boundary conditions. (See Jafri et al.)

$$\frac{\partial^2 w}{\partial t^2} = -\alpha^2 \frac{\partial^4 w}{\partial y^4} \tag{1}$$

$$w(0,t) = 0 \tag{2}$$

$$\left.\frac{\partial w}{\partial y}\right|_{(0,t)} = 0 \tag{3}$$

$$\left.\frac{\partial^2 w}{\partial y^2}\right|_{(1,t)} = 0 \tag{4}$$

$$\left.\frac{\partial^2 w}{\partial t^2}\right|_{(1,t)} = \frac{\alpha^2}{\mu} \left.\frac{\partial^3 w}{\partial y^3}\right|_{(1,t)} \tag{5}$$

(Here α and μ are dimensionless parameters. Both are positive constants.)

Problem Statement

1. Equation (1) has a solution in the form $w(y,t) = Y(y)T(t)$ provided the functions Y and T satisfy the ode's

$$T'' + \omega^2 T = 0 \tag{6}$$

and

$$Y^{(4)} - \beta^4 Y = 0. \tag{7}$$

Here ω is a positive parameter and $\beta = \sqrt{\omega/\alpha}$.
Find the general solutions of equations (6) and (7). Express the function Y in terms of the functions cos, sin, cosh, and sinh.

2. In order to satisfy boundary conditions (2) - (4), the function Y must satisfy the following conditions.

$$Y(0) = 0 \tag{8}$$
$$Y'(0) = 0 \tag{9}$$
$$Y''(1) = 0 \tag{10}$$

Use conditions (8) - (10) to show that

$$Y(y) = d_1 \left\{ \cos(\beta y) - \cosh(\beta y) - \left(\frac{\cos(\beta) + \cosh(\beta)}{\sin(\beta) + \sinh(\beta)} \right) [\sin(\beta y) - \sinh(\beta y)] \right\} \tag{11}$$

3. In order for boundary condition (5) to be satisfied, β must satisfy the equation

$$\mu\beta \left[\cosh(\beta) \sin(\beta) - \sinh(\beta) \cos(\beta) \right] - \left[1 + \cosh(\beta) \cos(\beta) \right] = 0. \tag{12}$$

Use MATLAB's *fzero* algorithm to find the smallest positive solution of equation (12). Take $\mu = 2.0817 \times 10^{-2}$.

4. **Find the value of the dimensionless frequency ω corresponding to the value of β you found in the previous step. Take $\alpha = 3.0992 \times 10^{-4}$.**

5. For each solution β_k of equation (12) there are a corresponding frequency ω_k and a corresponding solution $w_k(y, t) = Y_k(y)T_k(t)$ of equations (1) - (5). Unlike a simple undamped mass-spring system, which has just one natural frequency of oscillation, this system has multiple natural frequencies.

Under certain conditions, the rotating blades of the turbine can transmit a periodic force to the tower. The frequency of this force equals the frequency of rotation of the blades. As you know, resonance occurs in an undamped system when the frequency of the forcing term equals a natural frequency of oscillation of the system. In the problem under consideration, the smallest natural frequency is the one most likely to be close to the frequency of the forcing term.

According to Jafri et al., "the operating speed range of most of the commercial wind turbines [is] about 15 cpm - 30 cpm," which translates to a dimensionless frequency range of approximately 1.25×10^{-3} to 2.50×10^{-3} with the scaling used in this project.

Does the dimensionless frequency ω you found in the previous step lie in the range 1.25×10^{-3} to 2.50×10^{-3}? Do you expect resonance to occur with the parameter values used in this project?

Reference

Jafri, S. M., Eltaher, A., and Jukes, P,. "Dynamics of Offshore Wind Turbines," in **Proceedings of the Twenty-first (2011) International Offshore and Polar Engineering Conference**.

<div align="center">Grade Sheet</div>

1a. Solution of equation (6) (10 points) _____

1b. Solution of equation (7) (25 points) _____

2. Derivation of equation (11) (20 points) _____

3. Calculation of smallest root of equation (12) (25 points) _____

4. Calculation of dimensionless frequency (10 points) _____

5. Determination whether resonance is likely (10 points) _____

Total. _____

Answer Sheet

1a. $T(t)$ = _____

1b. $Y(y)$ = _____

3. β_1 = _____

4. ω_1 = _____

5. Do you expect resonance to occur? _____