92.236 Engineering Differential Equations Final Exam Fall 2011

Please write all answers and all work in the blue book provided.
PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work.

Problem 1. (10 pts.)

Solve the following initial value problem: $y' = \dfrac{1}{3y^2\,(2x+1)}$ with $y(0) = 1$. Note: y' means dy/dx.

Problem 2. (10 pts.)

Solve the following initial value problem: $y' = \dfrac{2x+y}{x}$, $y(1) = 2$.

Express your solution y explicitly in terms of x. In other words, write your answer in the form $y =$ something. Note: y' means dy/dx.

Problem 3. (15 points)

Let t denote time (in days) and let P denote the size of a mosquito population (in grams) at time t. Suppose the daily birth rate per gram is $\beta = 6 - 2P$, and suppose the daily death rate per gram is $\delta = 2$. (The units of β and δ are (gram/day)/gram.)

a. (6 pts.) Write down the differential equation modeling this problem ($\dfrac{dP}{dt} =$ something).

b. (6 pts.) Draw the phase line for the d.e. from part a.

c. (3 pts.) Suppose $P(0) = 1$. Use your phase line to find the limiting value of $P(t)$ as t increases.

Problem 4. (10 points)

Find the general solution to each of the following differential equations.

a. (4 points) $y'' + 2y' + 5y = 0$

b. (6 points) $y^{(4)} + 5y^{(3)} + 6y'' = 0$

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Problem 5. (15 points)

Solve the following initial value problem:

$$y'' - y' - 2y = 4x - 8e^{3x} \quad \text{with } y(0) = 2 \text{ and } y'(0) = -8.$$

Note: $y' = dy/dx$ and $y'' = d^2y/dx^2$

Problem 6. (15 points)

Consider a damped, forced mass/spring system. Let t denote time (in seconds) and let $x(t)$ denote the position (in meters) of the mass at time t, with $x = 0$ corresponding to the equilibrium position. Suppose the mass $m = 2$ kg, the damping constant $c = 8$ N·s/m, the spring constant $k = 6$ N/m, and the external force is $F_e(t) = 240\cos(3t)$.

a. (13 pts.) Find the steady-state (steady periodic) solution x_{sp}.

b. (2 pts.) Express your answer to part a in the form $x_{sp} = C\cos(\omega t - \alpha)$

Problem 7. (10 points)

a. (3 pts.) Find $\mathcal{L}\left\{\sqrt{t} + \sin(2t)\right\}$

b. (7 pts.) Find $\mathcal{L}^{-1}\left\{\dfrac{8}{s^2 - 6s + 25}\right\}$.

Problem 8. (15 points)

Use the Laplace Transform to solve the following initial value problem:

$$x'' + 4x = 10e^t \quad \text{with } x(0) = 0 \text{ and } x'(0) = 2.$$

Solutions to this IVP not using the Laplace transform method will not receive any credit. Primes denote derivatives with respect to t: $x' = dx/dt$ and $x'' = d^2x/dt^2$.