92.236 Engineering Differential Equations Final Exam Fall 2013

Please write all answers and all work in the blue book provided.
PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work.

Problem 1. (10 pts.)

Solve the following initial value problem: $x\dfrac{dy}{dx} = y + x^2, \quad y(1) = 0.$

Problem 2. (10 pts.)

Solve the following initial value problem: $\dfrac{dy}{dx} = \dfrac{2xy}{y^2+1}$ with $y(0) = 1.$

Problem 3. (15 points)

A cup of coffee at temperature 125° F is brought into a room where the temperature is 75° F. After 5 minutes the coffee temperature is 120° F. When will the coffee temperature reach 100° F?

Recall that the de modeling heating/cooling problems is $\dfrac{dT}{dt} = -k\,(T - A)$.

Problem 4. (10 points)

Solve each of the following differential equations.

a. (4 points) $y'' - 5y' + 6y = 0$

b. (6 points) $y^{(4)} + 2y^{(3)} + 2y'' = 0$

Problem 5. (15 points)

Solve the following initial value problem:

$$y'' + y' - 6y = 50\cos(x) \text{ with } y(0) = -5 \text{ and } y'(0) = 5.$$

Note: $y' = dy/dx$ and $y'' = d^2y/dx^2$

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Problem 6. (15 points) Find the position function $x(t)$ for an unforced, damped mass-spring system with mass 1 kg, damping coefficient 6 Ns/m, and spring constant 9 N/m. Take $x(0) = 1$ m and $x'(0) = 0$ m/s.

Problem 7. (10 points)

a. (2 pts.) Find $\mathcal{L}\left\{t^2 + \sin(3t)\right\}$

b. (8 pts.) Find $\mathcal{L}^{-1}\left\{\dfrac{4s+12}{(s+1)(s^2+1)}\right\}$.

Problem 8. (15 points)

Use the Laplace Transform to solve the following initial value problem:

$$x'' - x = 6e^{2t} \quad \text{with } x(0) = 0 \text{ and } x'(0) = 2.$$

Solutions to this IVP not using the Laplace transform method will not receive any credit. Primes denote derivatives with respect to t: $x' = dx/dt$ and $x'' = d^2x/dt^2$.