

92.236 Engineering Differential Equations Final Exam
Fall 2014

Problem 1. (15 pts.)

Solve the following initial value problem: $y' + y = x$, $y(0) = 1$.

Problem 2. (10 pts.)

Solve the following initial value problem: $xy' + y^2 = 2xy^2$, $y(1) = 1$.

Problem 3. (10 pts.)

A tank initially contains 100 grams of a radioactive substance. After 1 hour there are 90 grams of the substance remaining in the tank. What is the half-life of the substance? In other words, when will there be 50 grams of the substance remaining in the tank?

Problem 4. (10 pts.) Find the general solution to each of the following linear homogeneous differential equations:

a. (5 pts.) $y^{(4)} - 4y''' + 3y'' = 0$

b. (5 pts.) $y''' - 4y'' + 4y' = 0$

Problem 5. (15 pts.)

Solve the following initial value problem: $y'' + y' - 2y = 8x^2$, $y(0) = 4$, $y'(0) = 0$.

Problem 6. (15 points)

Consider a forced, damped mass-spring system with mass 1 kg, damping coefficient 2 Ns/m, spring constant 9 N/m, and an external force $F_{\text{ext}}(t) = 12 \cos(3t)$ N. Find the steady periodic solution (steady-state solution) for this system.

Problem 7. (10 points)

a. (3 pts.) Find the Laplace transform of $e^{-t} \cos(2t)$

b. (7 pts.) Find the inverse Laplace transform of $\frac{s+1}{s^2-3s+2}$.

Problem 8. (15 points)

Use the Laplace Transform to solve the following IVP: $x'' + x = 2e^t$, $x(0) = 1$, $x'(0) = 0$.

Solutions not using the Laplace transform method will not receive any credit.