MATH.2360 Engineering Differential Equations
Review Sheet for Exam #2

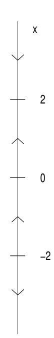
Necessary Skills

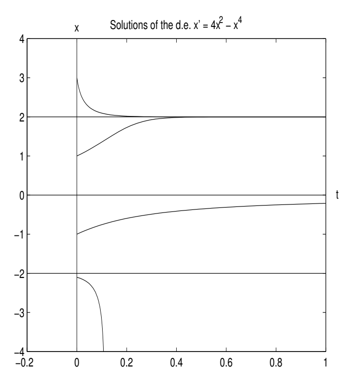
Section	You should
1.6	• be able to recognize and solve homogeneous first-order d.e.'s/initial value problems • be able to determine whether a first-order d.e. is exact • be able to solve exact first-order d.e.'s/initial value problems
2.1	• be able to formulate and solve first-order d.e.'s to analyze population problems
2.2	• be able to find the critical points of an autonomous first-order d.e. • be able to draw the phase line of an autonomous first-order d.e. • be able to determine the stability of critical points of an autonomous first-order d.e. • be able to sketch graphs of solutions of an autonomous first-order d.e. using the phase line • be able to determine the long-term behavior of solutions of an autonomous first-order d.e. using the phase line
2.3	• be able to formulate and solve first-order d.e.'s to analyze 1D motion of an object given information about the forces on the object or the object's acceleration
3.1, 3.2	• know that the general solution of a second-order linear homogeneous ode has the form $y = c_1y_1 + c_2y_2$, where y_1 and y_2 are independent solutions of the ode. • know that the general solution of a second-order linear nonhomogeneous ode has the form $y = y_c + y_p$ where y_c is the general solution of the corresponding homogeneous ode and y_p is a particular solution of the given nonhomogeneous ode • be able to find the values of the two arbitrary constants in the general solution of a second-order ode, given two initial conditions • be able to solve second-order linear homogeneous ode's with constant coefficients

Answers to Sample Problems

(Full solutions are available on the course web page under the Course Materials link.)

1. a. The critical points are $-2, 0$ and 2 c. 2 is stable but -2 and 0 are unstable
d. $x(t) \to 0$ as t increases. b. and e.





2. a. $y = c_1e^{5x} + c_2xe^{5x}$ b. $y = c_1e^{-2x} + c_2e^{x}$ c. $y = c_1e^{-4x} + c_2e^{-2x}$ d. $y = c_1e^{3x} + c_2xe^{3x}$
3. $x^3 + 3x^2y + y^2 = 1$ 4. $y = x\left[3\ln(x) + 27\right]^{1/3}$ 5. $y = x\left[\ln(x) + 16\right]^{1/4}$
6. $x^2y^2 + x^3 + y^4 = 21$ 7. $P(43) \approx 5$ dodos 8. $\dfrac{dv}{dt} = -g + k_1v^2$

MATH.2360 Engineering Differential Equations   Sample Problems for Exam # 2

There is no guarantee that the questions on the actual exam will bear any resemblance to these sample problems.

Problem 1. Consider the autonomous differential equation $\dfrac{dx}{dt} = 4x^2 - x^4$.

a. Find all critical points (equilibrium solutions) of this d.e.

b. Draw the phase line (phase diagram) for this d.e.

c. Determine whether each critical point is stable or unstable.

d. If $x(0) = -1$, what value will $x(t)$ approach as t increases?

e. Sketch typical solution curves of the given d.e. Please draw these curves in a graph separate from your phase line. Be sure to include graphs of all equilibrium solutions, and be sure to label the axes.

Problem 2. Solve the following differential equations:

a) $y'' - 10y' + 25y = 0$ b) $y'' + y' - 2y = 0$

c) $y'' + 6y' + 8y = 0$ d) $y'' - 6y' + 9y = 0$

Solve the following initial value problems.

Problem 3. $3x^2 + 6xy + \left[3x^2 + 2y\right] \frac{dy}{dx} = 0, \quad y(1) = 0.$

Problem 4. $x^3 + y^3 - xy^2 \frac{dy}{dx} = 0, \quad y(1) = 3.$

Problem 5. $x^4 + 4y^4 - 4xy^3 \frac{dy}{dx} = 0, \quad y(1) = 2.$

Problem 6. $2xy^2 + 3x^2 + \left(2x^2 y + 4y^3\right) \frac{dy}{dx} = 0, \quad y(1) = 2.$

Problem 7. Let P denote the population of a colony of dodos. Suppose that the birth rate β (number of births per week per dodo) equals 0 and that the death rate δ (number of deaths per week per dodo) is constant. Suppose the initial population is 100, and after ten weeks the population is 50. What is the population after 43 weeks?

Problem 8. A ball of mass m falling vertically downward experiences two forces: its weight, and a drag force proportional to the *square* of its velocity. Let t denote time, and let v denote the ball's velocity at time t. (Assume $v < 0$ for the falling object. In other words, up is the positive direction.)

Write a differential equation (dv/dt = something) modeling the ball's motion. Make sure the drag force has the proper sign (+ or −). All parameters are assumed to be positive.

DO NOT SOLVE THE DIFFERENTIAL EQUATION, JUST WRITE IT DOWN.