MATH.2360 Engineering Differential Equations
Review Sheet for Exam #3

Section	You should
3.3	• be able to solve n^{th} order linear homogeneous ode's with constant coefficients
3.4	• be able to formulate and solve the second-order linear homogeneous ode describing the unforced motion of a mass attached to a spring: $mx'' + cx' + kx = 0$ • be able to express the solution to an undamped mass/spring problem in the form $x = C\cos(\omega_0 t - \alpha)$ • be able to tell whether a system is overdamped, underdamped, or critically damped • be able to express the solution to an underdamped mass/spring problem in the form $x = Ce^{-pt}\cos(\omega_1 t - \alpha)$
3.5	• be able to find a particular solution of a nonhomogeneous linear equation using either the Method of Undetermined Coefficients or the Method of Variation of Parameters
3.6	• be able to formulate and solve the second-order linear nonhomogeneous ode describing the forced motion of a mass attached to a spring: $mx'' + cx' + kx = F(t)$ • be able to find the steady-state periodic solution and the transient solution of a damped, forced mass-spring system
3.7	• be able to formulate and solve the second-order linear nonhomogeneous ode describing the forced motion of an LCR circuit: $LQ'' + RQ' + \frac{1}{C}Q = E(t)$
4.1	• be able to solve systems of 2 linear constant coefficient d.e.'s

Answers to Practice Exam Questions

(Full solutions are available on the course web page under the Course Materials link.)

1a. $y = c_1e^{-x}\cos(x) + c_2e^{-x}\sin(x)$ 1b. $y = c_1 + c_2e^{5x} + c_3xe^{5x}$
1c. $y = c_1e^{3x}\cos(4x) + c_2e^{3x}\sin(4x)$ 1d. $y = c_1 + c_2x + c_3e^{-2x} + c_4xe^{-2x}$
2. $y = 8e^{-x} + 4x - 8 + 2e^x$
3. $y = -6 + 6\cos(x) - 3\sin(x)$
4. $I_{\text{sp}} = 16\cos(4t)$
5a. $x = -e^{-3t}\cos(t) - 3e^{-3t}\sin(t)$ 5b. $x = \sqrt{10}e^{-3t}\cos\left(t - \left(\pi + \tan^{-1}(3)\right)\right)$
5c. underdamped
6. $x = -\frac{1}{3}c_1e^{-t} + c_2e^{3t},\ y = c_1e^{-t} + c_2e^{3t}$
7. $x = c_1e^{-t} + c_2e^{-3t},\ y = -c_1e^{-t} - 3c_2e^{-3t}$

There is no guarantee that the actual exam will bear any resemblance to these sample problems.

Problem 1. Solve the following differential equations.

a. $y'' + 2y' + 2y = 0$.

b. $y^{(3)} - 10y'' + 25y' = 0$.

c. $y'' - 6y' + 25y = 0$.

d. $y^{(4)} + 4y^{(3)} + 4y'' = 0$.

Problem 2. Solve the following initial value problem:

$$y'' + 2y' + y = 4x + 8e^x, \; y(0) = 2, \; y'(0) = -2.$$

Problem 3. Solve the following initial value problem:

$$y'' - 2y' = 15\sin(x), \; y(0) = 0, \; y'(0) = -3.$$

Problem 4. Consider an RLC circuit with inductance $H = 1$ henry, resistance $R = 2\Omega$, capacitance $C = 1/16$ farad, and applied voltage $E(t) = 32\cos(4t)$ volts. Find the steady periodic current $I_{sp}(t)$.

Problem 5. Consider a free (unforced), damped mass-spring system with mass $m = 1$ kg, damping constant $c = 6$ N·s/m, and spring constant $k = 10$ N/m. Assume that $x(0) = -1$ and $x'(0) = 0$.

a. Find the position function $x(t)$.

b. Express your solution from part a in the form $x = Ce^{-pt}\cos(\omega_1 t - \alpha)$

c. Is this system overdamped, underdamped, or critically damped?

Problem 6. Solve the system $\begin{cases} x' &= 2x + y \\ y' &= 3x \end{cases}$

Note: $x' = dx/dt$ and $y' = dy/dt$. t is the independent variable.

Problem 7. Solve the system $\begin{cases} x' &= y \\ y' &= -3x - 4y \end{cases}$

Note: $x' = dx/dt$ and $y' = dy/dt$. t is the independent variable.