92.236 Engineering Differential Equations Review Sheet for Final Exam

Policy on Time Conflicts
If you have 2 exams scheduled at the same time, or if you have 3 exams scheduled on the same day, that is considered a time conflict, and you can have one of the exams rescheduled. Required courses take precedence over elective courses when determining which exam will be rescheduled.

Necessary Skills

I. First Order D.E.'s

A. Analytical Techniques. You should be able to recognize and solve the following types of first order d.e.'s:

- separable
- linear
- homogeneous
- exact

B. Qualitative Techniques. You should be able to

- find the critical points of an autonomous first-order d.e.
- draw the phase line and solution curves of an autonomous first-order d.e.
- determine the stability of critical points of an autonomous first-order d.e.
- determine the long-term behavior of solutions of an autonomous first-order d.e. using the phase line

C. Applications. You should be able to formulate and solve first order d.e.'s to analyze the following types of problem:

- radioactive decay
- compound interest
- cooling/heating
- mixture
- population models
- 1D motion of an object (given information about the forces on the object or the object's acceleration)

II. Higher Order Linear D.E.'s

A. Analytical Techniques. You should

- know that the general solution of an n^{th} order linear homogeneous d.e. has the form $y = c_1y_1 + c_2y_2 + \dots c_ny_n$, where y_1, y_2, $\dots$ y_n are independent solutions of the d.e.
- know that the general solution of an n^{th} order linear nonhomogeneous d.e. has the form $y = y_c + y_p$ where y_c is the general solution of the corresponding homogeneous d.e. and y_p is a particular solution of the given nonhomogeneous d.e.
- be able to solve n^{th} order linear homogeneous d.e.'s with constant coefficients
- be able to find a particular solution of a nonhomogeneous linear equation using either the Method of Undetermined Coefficients or the Method of Variation of Parameters
- be able to find the values of the arbitrary constants in the general solution of an n^{th} order d.e., given n initial conditions

B. Applications.

1. Mass-spring systems. You should
   - be able to formulate and solve the second-order linear homogeneous d.e. describing the motion (forced or unforced, damped or undamped) of a mass attached to a spring: $mx'' + cx' + kx = F(t)$
   - be able to rewrite the expression $c_1 \cos(\omega t) + c_2 \sin(\omega t)$ in the form $C \cos(\omega t - \alpha)$
   - be able to tell whether a system is overdamped, underdamped, or critically damped
   - be able to find the steady-state periodic solution and the transient solution of a damped, periodically forced mass-spring system
   - be able to find the period and frequency of a sinusoidal function
2. LRC circuits. You should
   - be able to formulate and solve the second-order linear nonhomogeneous d.e. describing the forced motion of an LRC circuit:$LQ'' + RQ' + \frac{1}{C}Q = E(t)$
   - be able to find the steady-state periodic solution and the transient solution of an LRC circuit.

III. Systems of First Order D.E.'s. You should be able to

- transform an n^{th} order d.e. into a system of n first order d.e.'s
- solve systems of 2 linear constant coefficient d.e.'s
- formulate systems of equations describing multi-tank mixture problems and coupled mass-spring systems

IV. Laplace Transforms. You should be able to

- find the Laplace transform of a given function using the definition
- find the Laplace transform of a given function using the tables
- find the inverse Laplace transform of a given function using the tables, partial fraction decomposition, and/or completing the square
- solve initial value problems using the Laplace transform method

V. General. You should be able to

- determine the order of a given d.e., and you should know that the number of arbitrary constants in the general solution of a d.e. equals the order
- determine whether a given function is a solution of a given d.e.
- translate a verbal description of a physical system into a d.e.

92.236 Engineering Differential Equations Sample Problems

There is no guarantee that the actual exam will bear any resemblance to these sample problems.

Problem 1. Solve the following initial value problem: $xy' - \frac{y^2}{x^2} = 0,\ y(1) = 1.$

Problem 2. Solve the following initial value problem: $xyy' + y^2 - x^2 = 0,\ y(2) = 1.$

Problem 3. Solve the following initial value problem: $y' - \frac{4y}{x} = x^4 \cos(x),\ y(\pi) = 0.$

Problem 4. Solve the following initial value problem: $2xyy' + y^2 - 4x^3 = 0,\ y(1) = 2.$

Problem 5. Let P denote the population of a colony of tribbles. Suppose that β (the number of births per week per tribble) is proportional to $\sqrt{P}$ and that δ (the number of deaths per week per tribble) equals 0. Suppose the initial population is 4 and the population after 1 week is 9. What is the population after 2 weeks?

Recall that the de modeling population problems is $\frac{dP}{dt} = \beta P - \delta P.$

Problem 6. Find the general solution to each of the following linear homogeneous differential equations:

a. $y^{(3)} + 2y'' + 2y' = 0$
b. $y^{(4)} - 9y'' = 0$

Problem 7. Consider a forced, damped mass-spring system with mass 1 kg, damping coefficient 2 Ns/m, spring constant 4 N/m, and an external force $F_{\text{ext}}(t) = 8\cos(2t)$ N. Find the steady-state periodic solution $x_{\text{sp}}(t)$.

Problem 8. Consider an RLC circuit with inductance $L = 1$ henry, resistance $R = 5\Omega$, capacitance $C = 0.25$ farads, and applied voltage $E(t) = 20\cos(2t)$ volts. Suppose the initial charge on the capacitor $Q(0) = 1$ coul and the initial current in the circuit $Q'(0) = 0$ amps. Find the current in the circuit $I(t)$.

Problem 9. Use the Laplace Transform to solve the following IVP: $x'' + 5x' + 6x = 4e^{-t},\ x(0) = 1,\ x'(0) = 0.$

Solutions not using the Laplace transform method will not receive any credit. x is a function of t. x'' means $\frac{d^2x}{dt^2}$.

Answers to Practice Exam Questions

(Full solutions are available on the course web page under the Course Materials link.)

1. $y = \dfrac{2x^2}{x^2+1}$

2. $y = \dfrac{\sqrt{x^4-8}}{\sqrt{2}x}$

3. $y = x^4 \sin(x)$

4. $y = \sqrt{\dfrac{x^4+3}{x}}$

5. $P(2) = 36$ tribbles

6a. $y = c_1 + c_2 e^{-x}\cos(x) + c_3 e^{-x}\sin(x)$

6b. $y = c_1 + c_2 x + c_3 e^{-3x} + c_4 e^{3x}$

7. $x_{\text{sp}} = 2\sin(2t)$ meters

8. $I(t) = 4\cos(2t) - 4e^{-4t}$ amps

9. $x = 2e^{-t} - e^{-2t}$