

92.236 Engineering Differential Equations
Take-Home Part of Exam # 3
Fall 2016

Due date: Friday, November 18.

Problem #1 (15 points)

The purpose of this assignment is to illustrate one important difference between linear and nonlinear models of oscillating systems. In an undamped, unforced **linear** system, the period of a periodic solution depends only on system parameters and not on the initial conditions, but in a **nonlinear** system the period can depend on the initial conditions.

Consider the following nonlinear differential equation, which models the free, undamped motion of a block attached to a “soft” spring. (A soft spring is a spring which requires less force to stretch than a spring that obeys Hooke’s Law.)

$$x'' + x - 0.25x^3 = 0. \tag{1}$$

- a. Transform the second-order d.e. above into an equivalent system of first-order d.e.’s. Note: x^3 means x raised to the third power, **not** the third derivative of x .
- b. Use MATLAB’s ode45 solver to generate a numerical solution of this system over the interval $0 \leq t \leq 12\pi$ for the following two sets of initial conditions.
 - i. $x(0) = 0.1, x'(0) = 0$
 - ii. $x(0) = 1, x'(0) = 0$
- c. Graph the two solutions on the same set of axes. **Graph only x vs. t for each IVP; do not graph x' . Use the plot command, not the plotyy command. Be sure to label the axes.** Include a title that contains your name and describes the graph, something like “Numerical Solutions of $x'' + x - 0.2x^3 = 0$ by I. M. Smart.”

Note: To get x'' to appear in your title you will have to type x'''' in your MATLAB title command.
- d. Based on your graph, which solution appears to have the longer period?

Please turn in your answers to parts a and d, your graph from part c (*which should only contain 2 curves, not 4*), and your MATLAB code including the m file defining the system of differential equations. I don’t want to see all the calculated values. Please email your results to me at stephen_pennell@uml.edu