92.236 Engineering Differential Equations Final Exam Spring 2012

Please write all answers and all work in the blue book provided.
PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work.

Problem 1. (10 pts.)

Solve the following initial value problem: $y' = \dfrac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = -1.$

Note: y' means dy/dx.

Problem 2. (10 pts.)

Solve the following initial value problem: $2xy^2 + 4 = \left(6 - 2x^2y\right) y' \quad \text{with} \quad y(-1) = 8.$

Note: y' means dy/dx.

Problem 3. (15 points)

A tank initially contains 12 liters of pure water (no salt). Water containing 2 grams of salt per liter is pumped into the tank at the rate of 3 liters per minute, and the well-mixed solution in the tank is pumped out of the tank at the rate of 3 liters per minute. How long will it take for the amount of salt in the tank to reach 12 grams?

Problem 4. (10 points)

Find the general solution to each of the following differential equations.

a. (4 points) $y'' - 4y' + 5y = 0$

b. (6 points) $y^{(3)} + 4y'' + 4y' = 0$

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Problem 5. (15 points)

Solve the following initial value problem:

$$y'' - 5y' + 4y = 32x + 20\sin(2x) \text{ with } y(0) = 10 \text{ and } y'(0) = 0.$$

Note: $y' = dy/dx$ and $y'' = d^2y/dx^2$

Problem 6. (10 points)

A series circuit consists of a generator supplying $120\cos(4t)$ volts, a 5 ohm resistor, and a 0.05 farad capacitor. Assume the initial charge on the capacitor is 0. Find the charge on the capacitor $Q(t)$.

Remember that the d.e. modeling the charge on the capacitor in an RC circuit is $RQ' + \dfrac{Q}{C} = E(t)$

Problem 7. (15 points)

a. (4 pts.) Find $\mathcal{L}\left\{e^{-2t}\cos(3t) - 4t^2\right\}$

b. (5 pts.) Find $\mathcal{L}\{f(t)\}$ where

$$f(t) = \begin{cases} \sin(t) & \text{if } t < 2\pi \\ 0 & \text{if } t \geq 2\pi \end{cases}$$

c. (6 pts.) Find $\mathcal{L}^{-1}\left\{\dfrac{2s+4}{s^2+4s-5}\right\}$.

Problem 8. (15 points)

Use the Laplace Transform to solve the following initial value problem:

$$x'' + 4x' + 8x = 0 \text{ with } x(0) = 0 \text{ and } x'(0) = 2.$$

Solutions to this IVP not using the Laplace transform method will not receive any credit. Primes denote derivatives with respect to t: $x' = dx/dt$ and $x'' = d^2x/dt^2$.