92.236 Engineering Differential Equations Final Exam
Spring 2014

Problem 1. (5+5 pts.)

The first-order differential equation $xy' = -y$ is separable, linear, exact, and homogeneous. Find its general solution **by using any two different of the above four methods**.

Problem 2. (15 pts.)

Solve the following initial value problem (IVP) for $x > 0$: $x^2y' - xy = 2y^2$ with $y(1) = -1$.

Problem 3. (10 pts.) Consider the expansion-contraction equation: $\dfrac{dP}{dt} = P(10 - P)(P - 20)$.

a. Find the critical points of this equation and sketch the phase line. Determine the stability of the critical points and mark the results on the phase line, along with arrows that indicate the sign of the derivative $\dfrac{dP}{dt}$.

b. For each of the initial conditions $P(0) = 5$, $P(0) = 15$, and $P(0) = 25$, state where the corresponding particular solutions are headed as $t \to \infty$, that is, find $\lim\limits_{t\to\infty} P(t)$.

Problem 4. (15 points)

A 20-liter tank initially contains 10 liters of water in which 5 grams of salt are dissolved. A solution containing 20 grams of salt per liter is pumped into the tank at the rate of 2 liters per minute, and the well-mixed solution is pumped out of the tank at the rate of 1 liter per minute. How much salt will the tank contain when it is full?

Problem 5. (5+5 pts.) Find the general solution to each of the following third-order linear homogeneous differential equations:

a. $y''' - 8y'' + 16y' = 0$

b. $y''' - y' = 0$

Problem 6. (15 points)

Consider a forced, undamped mass-spring system with mass $m = 1$ kg, spring constant $k = 9$ N/m, and an external force $F_{\text{ext}}(t) = 10\cos(2t)$ N. Find the position function $x(t)$ for the initial conditions $x(0) = 2$ and $x'(0) = 3$.

Problem 7. (5+5 points) Find the inverse Laplace transforms of the following two functions:

a. $\dfrac{s+2}{s^2+4}$

b. Find $\dfrac{s}{(s+2)^2+4}$.

Problem 8. (15 points)

Use the Laplace Transform to solve the following IVP: $x'' - x' = 1; x(0) = 2; x'(0) = 1$

Solutions not using the Laplace transform method will not receive any credit.