92.236 Engineering Differential Equations Final Exam
Spring 2015

Problem 1. (15 pts.)

Solve the following initial value problem: $y' = \dfrac{2y + x^2}{x},\ y(1) = 2.$

Problem 2. (10 pts.)

Solve the following initial value problem: $y' = \dfrac{xy + y^2}{x^2},\ y(1) = -1.$

Problem 3. (10 pts.)

A tank initially contains 50 liters of water in which 250 grams of salt are dissolved. Water containing 10 grams per liter of salt in pumped into the tank at the rate of 5 liters per minute, and the well-mixed solution is pumped out of the tank at the rate of 5 liters per minute. How many grams of salt will be in the tank after 10 minutes?

Problem 4. (10 pts.) Find the general solution to each of the following linear homogeneous differential equations:

a. (5 pts.) $y''' + 4y' = 0$

b. (5 pts.) $y^{(4)} + 2y''' + 5y'' = 0$

Problem 5. (15 pts.)

Solve the following initial value problem: $y'' - y = 4e^x,\ y(0) = 1,\ y'(0) = 1.$

Problem 6. (15 points)

Consider a forced, undamped mass-spring system with mass 1 kg, damping coefficient 0 Ns/m, spring constant 9 N/m, and an external force $F_{\text{ext}}(t) = 10\cos(2t)$ N. Suppose the initial position and velocity of the mass are given by $x(0) = 2$ m and $x'(0) = 3$ m/s. Find the position function $x(t)$.

Problem 7. (10 points)

a. (3 pts.) Find the Laplace transform of te^{2t}

b. (7 pts.) Find the inverse Laplace transform of $\dfrac{2}{s^3 + s}$.

Problem 8. (15 points)

Use the Laplace Transform to solve the following IVP: $x'' - 4x = 8t,\ x(0) = 1,\ x'(0) = 0.$

Solutions not using the Laplace transform method will not receive any credit. x is a function of t. x'' means $\dfrac{d^2x}{dt^2}$.