

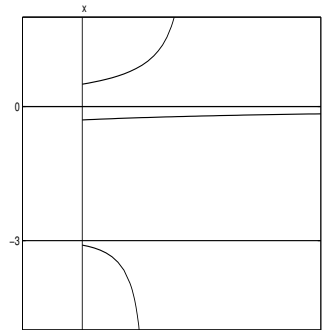
Necessary Skills

Section	You should
1.6	<ul style="list-style-type: none"> be able to recognize and solve homogeneous first-order d.e.'s/initial value problems be able to determine whether a first-order d.e. is exact be able to solve exact first-order d.e.'s/initial value problems
2.1	<ul style="list-style-type: none"> be able to formulate and solve first-order d.e.'s to analyze population problems
2.2	<ul style="list-style-type: none"> be able to find the critical points of an autonomous first-order d.e. be able to draw the phase line of an autonomous first-order d.e. be able to determine the stability of critical points of an autonomous first-order d.e. be able to sketch graphs of solutions of an autonomous first-order d.e. using the phase line be able to determine the long-term behavior of solutions of an autonomous first-order d.e. using the phase line
2.3	<ul style="list-style-type: none"> be able to formulate and solve first-order d.e.'s to analyze 1D motion of an object given information about the forces on the object or the object's acceleration
3.1, 3.2	<ul style="list-style-type: none"> know that the general solution of a second-order linear homogeneous ode has the form $y = c_1y_1 + c_2y_2$, where y_1 and y_2 are independent solutions of the ode. know that the general solution of a second-order linear nonhomogeneous ode has the form $y = y_c + y_p$ where y_c is the general solution of the corresponding homogeneous ode and y_p is a particular solution of the given nonhomogeneous ode be able to find the values of the two arbitrary constants in the general solution of a second-order ode, given two initial conditions be able to solve second-order linear homogeneous ode's with constant coefficients

Answers to Practice Exam Questions

(Full solutions are available on the course web page under the Course Materials link.)

1. a. The critical points are -3 and 0 c. Both -3 and 0 are unstable
d. $x(t) \rightarrow 0$ as t increases. b. and e.



2. a. $y = c_1e^{-5x} + c_2e^{-3x}$ b. $y = c_1e^{-3x} + c_2xe^{-3x}$

3. $y = x [16 - \ln(x)]^{1/4}$ 4. $x^2y + x^4 + 2y^3 = 4$

5. 9 weeks

There is no guarantee that the actual exam will bear any resemblance to this practice exam. The purpose of the practice exam is to give you an idea of the approximate length and the type of problem that you can expect on the actual exam.

Problem 1. (20 points) Consider the autonomous differential equation $\frac{dx}{dt} = x^3 + 3x^2$.

- Find all critical points (equilibrium solutions) of this d.e.
- Draw the phase line (phase diagram) for this d.e.
- Determine whether each critical point is stable or unstable.
- If $x(0) = -2$, what value will $x(t)$ approach as t increases?
- Sketch typical solution curves of the given d.e. Please draw these curves in a graph separate from your phase line. Be sure to include graphs of all equilibrium solutions, and be sure to label the axes.

Problem 2. (20 points) Solve the following differential equations:

a. $y'' + 8y' + 15y = 0$

b. $y'' + 6y' + 9y = 0$

Problem 3. (20 points) Solve the following initial value problem.

$$x^4 - 4y^4 + 4xy^3 \frac{dy}{dx} = 0, \quad y(1) = 2.$$

Problem 4. (20 points) Solve the following initial value problem.

$$2xy + 4x^3 + [x^2 + 6y^2] \frac{dy}{dx} = 0, \quad y(-1) = 1.$$

Problem 5. (10 points) Let P denote the population of a colony of tribbles. Suppose that β (the number of births per week per tribble) is proportional to P and that δ (the number of deaths per week per tribble) equals 0. Suppose the initial population is 2 and the population after 5 weeks is 4. When will the population reach 20?