92.236 Engineering Differential Equations Spring 2015
Review Sheet for Exam #3

Section	You should
3.3	• be able to solve n^{th} order linear homogeneous ode's with constant coefficients
3.4	• be able to formulate and solve the second-order linear homogeneous ode describing the unforced motion of a mass attached to a spring: $mx'' + cx' + kx = 0$ • be able to express the solution to an undamped mass/spring problem in the form $x = C\cos(\omega_0 t - \alpha)$ • be able to tell whether a system is overdamped, underdamped, or critically damped • be able to express the solution to an underdamped mass/spring problem in the form $x = Ce^{-pt}\cos(\omega_1 t - \alpha)$
3.5	• be able to find a particular solution of a nonhomogeneous linear equation using either the Method of Undetermined Coefficients or the Method of Variation of Parameters
3.6	• be able to formulate and solve the second-order linear nonhomogeneous ode describing the forced motion of a mass attached to a spring: $mx'' + cx' + kx = F(t)$ • be able to find the steady-state periodic solution and the transient solution of a damped, forced mass-spring system
3.7	• be able to formulate and solve the second-order linear nonhomogeneous ode describing the forced motion of an LCR circuit: $LQ'' + RQ' + \frac{1}{C}Q = E(t)$
4.1	• be able to transform an n^{th} order d.e. into a system of n first order d.e.'s • be able to solve systems of 2 linear constant coefficient d.e.'s • be able to formulate systems of equations describing multi-tank mixture problems and coupled mass-spring systems

Answers to Practice Exam Questions

(Full solutions are available on the course web page under the Course Materials link.)

1a. $y = c_1 e^{-2x}\cos(2x) + c_2 e^{-2x}\sin(2x)$ 1b. $y = c_1 + c_2 x + c_3\cos(2x) + c_4\sin(2x)$

2. $y = 2 + 3x - 2\sin(x) - 2\cos(x)$

3a. $x = -e^{-2t}\cos(t) + e^{-2t}\sin(t)$ 3b. $x = \sqrt{2}e^{-2t}\cos\left(t - \frac{3\pi}{4}\right)$

4. $x = c_1 e^t + c_2 t e^t$, $y = c_1 e^t + c_2\left[e^t + te^t\right]$

There is no guarantee that the actual exam will bear any resemblance to this practice exam. The purpose of the practice exam is to give you an idea of the approximate length and the type of problem that you can expect on the actual exam.

Problem 1. (20 pts.) Solve the following differential equations.

a. (8 pts.) $y'' + 4y' + 8y = 0$

b. (12 pts.) $y^{(4)} + 4y'' = 0$

Problem 2. (25 pts.) Solve the following initial value problem:

$$y'' + y' = 3 + 4\sin(x), \quad y(0) = 0, \quad y'(0) = 1.$$

Problem 3. (20 points) Consider a free (unforced), damped mass-spring system with mass $m = 1$ kg, damping constant $c = 4$ N·s/m, and spring constant $k = 5$ N/m. Assume that $x(0) = -1$ m and $x'(0) = 3$ m/s.

a. (16 pts.) Find the position function $x(t)$.

b. (4 pts.) Write the solution from part a in the form $x(t) = Ce^{-pt}\cos(\omega_1 t - \alpha)$

Problem 4. (20 points) Solve the system $\begin{cases} x' &= y \\ y' &= -x + 2y \end{cases}$

Note: $x' = dx/dt$ and $y' = dy/dt$. t is the independent variable.