MATH.2360 Engineering Differential Equations
Review Sheet for Exam #3

Section	You should
3.1, 3.2	• know that the general solution of a second-order linear homogeneous ode has the form $y = c_1y_1 + c_2y_2$, where y_1 and y_2 are independent solutions of the ode. • know that the general solution of a second-order linear nonhomogeneous ode has the form $y = y_c + y_p$ where y_c is the general solution of the corresponding homogeneous ode and y_p is a particular solution of the given nonhomogeneous ode • be able to find the values of the two arbitrary constants in the general solution of a second-order ode, given two initial conditions • be able to solve second-order linear homogeneous ode's with constant coefficients
3.3	• be able to solve n^{th} order linear homogeneous ode's with constant coefficients
3.4	• be able to formulate and solve the second-order linear homogeneous ode describing the unforced motion of a mass attached to a spring: $mx'' + cx' + kx = 0$ • be able to express the solution to an undamped mass/spring problem in the form $x = C\cos(\omega_0 t - \alpha)$ • be able to tell whether a system is overdamped, underdamped, or critically damped • be able to express the solution to an underdamped mass/spring problem in the form $x = Ce^{-pt}\cos(\omega_1 t - \alpha)$
3.5	• be able to find a particular solution of a nonhomogeneous linear equation using either the Method of Undetermined Coefficients or the Method of Variation of Parameters
3.6	• be able to formulate and solve the second-order linear nonhomogeneous ode describing the forced motion of a mass attached to a spring: $mx'' + cx' + kx = F(t)$ • be able to find the steady-state periodic solution and the transient solution of a damped, forced mass-spring system
3.7	• be able to formulate and solve the second-order linear nonhomogeneous ode describing the forced motion of an LCR circuit: $LQ'' + RQ' + \frac{1}{C}Q = E(t)$

Answers to Practice Exam Questions

(Full solutions are available on the course web page under the Course Materials link.)

1a. $y = c_1e^{-x}\cos(x) + c_2e^{-x}\sin(x)$ 1b. $c_2e^{5x} + c_2xe^{5x}$ 1c. $y = c_1 + c_2e^{5x} + c_3xe^{5x}$
1d. $y = c_1e^{-2x} + c_2e^{x}$ 1e. $y = c_1e^{3x}\cos(4x) + c_2e^{3x}\sin(4x)$ 1f. $y = c_1 + c_2x + c_3e^{-2x} + c_4xe^{-2x}$
2. $y = 8e^{-x} + 4x - 8 + 2e^{x}$
3. $y = -6 + 6\cos(x) - 3\sin(x)$
4. $I_{\text{sp}} = 16\cos(4t)$
5a. $x = -e^{-3t}\cos(t) - 3e^{-3t}\sin(t)$ 5b. $x = \sqrt{10}e^{-3t}\cos\left(t - \left(\pi + \tan^{-1}(3)\right)\right)$
5c. underdamped

There is no guarantee that the actual exam will bear any resemblance to these sample problems.

Problem 1. Solve the following differential equations.

a. $y'' + 2y' + 2y = 0.$ b. $y'' - 10y' + 25y = 0$

c. $y^{(3)} - 10y'' + 25y' = 0.$ d. $y'' + y' - 2y = 0$

e. $y'' - 6y' + 25y = 0.$ f. $y^{(4)} + 4y^{(3)} + 4y'' = 0.$

Problem 2. Solve the following initial value problem:

$$y'' + 2y' + y = 4x + 8e^x, \; y(0) = 2, \; y'(0) = -2.$$

Problem 3. Solve the following initial value problem:

$$y'' - 2y' = 15\sin(x), \; y(0) = 0, \; y'(0) = -3.$$

Problem 4. Consider an RLC circuit with inductance $H = 1$ henry, resistance $R = 2\Omega$, capacitance $C = 1/16$ farad, and applied voltage $E(t) = 32\cos(4t)$ volts. Find the steady periodic current $I_{sp}(t)$.

Problem 5. Consider a free (unforced), damped mass-spring system with mass $m = 1$ kg, damping constant $c = 6$ N·s/m, and spring constant $k = 10$ N/m. Assume that $x(0) = -1$ and $x'(0) = 0$.

a. Find the position function $x(t)$.

b. Express your solution from part a in the form $x = Ce^{-pt}\cos(\omega_1 t - \alpha)$

c. Is this system overdamped, underdamped, or critically damped?