MATH.2360 Engineering Differential Equations
Take-Home Part of Exam # 3
Spring 2019

Due date: Friday, April 19.

Problem #1 (15 points)

The purpose of this assignment is to illustrate one important difference between linear and nonlinear models of oscillating systems. In an undamped, unforced **linear** system, the period of a periodic solution depends only on system parameters and not on the initial conditions, but in a **nonlinear** system the period can depend on the initial conditions.

Consider the following nonlinear differential equation, which models the free, undamped motion of a block attached to a "soft" spring. (A soft spring is a spring which requires less force to stretch than a spring that obeys Hooke's Law.)

$$x'' + x - 0.1x^3 = 0. \tag{1}$$

a. Transform the second-order d.e. above into an equivalent system of first-order d.e.'s. Note: x^3 means x raised to the third power, **not** the third derivative of x.

b. Use MATLAB's ode45 solver to generate a numerical solution of this system over the interval $0 \le t \le 10\pi$ for the following two sets of initial conditions.

   i. $x(0) = 1,\ x'(0) = 0$

   ii. $x(0) = 2,\ x'(0) = 0$

c. Graph the two solutions on the same set of axes. **Graph only x vs. t for each IVP; do not graph x'. Do not use the plotyy command. Be sure to label the axes.** Include a title that contains your name and describes the graph, something like Numerical Solutions of $x'' + x - 0.1x^3 = 0$ by I. M. Smart.

   Note: To get x'' to appear in your title you will have to type x'''' in your MATLAB title command.

d. Based on your graph, which solution appears to have the longer period?

Please turn in your answers to parts a and d, your graph from part c (*which should only contain 2 curves, not 4*), and your MATLAB code including the m file defining the system of differential equations. I don't want to see all the calculated values. Please email your results to me at `stephen_pennell@uml.edu`