A. Basic Statistics

One of the purposes of statistics is to characterize a set of data. Suppose we have a set of real numbers $\{x_1, x_2, x_3, \ldots, x_n\}$. The *mean*, *median*, and *mode* of the data set are three different representative values of the set. The mean μ is defined to be

$$\mu = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

To find the median first sort the numbers. If n is odd, the median is the middle value of the sorted array; if n is even, the median is the average of the two middle values.

The mode is the most frequently occurring number in the set.

The *standard deviation* is a measure of the spread in the data. The standard deviation σ is defined to be

$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \ldots + (x_n - \mu)^2}{n - 1}}$$

A small value of σ means that the data are clustered near the mean, while a large value of σ means there is a wide spread in the data.

If x is defined as an array of numbers in MATLAB, mean(x) gives the mean value of the numbers, std(x) gives the standard deviation, median(x) gives the median, and mode(x) gives the mode. If multiple x values occur the same number of times, mode returns the smallest such value.

B. Working with Polynomials

A polynomial is represented in MATLAB by a row vector containing the coefficients of the polynomial, starting with the highest power of x. For example, the polynomial $f(x) = x^2 - 5x + 6$ would be represented by the vector p = [1 -5 6]. (You can use any valid MATLAB name you like; you do not have to use the letter p.)

To evaluate a polynomial, you do not have to write a separate function file for every polynomial; you can use the commmand polyval. For example, to evaluate $f(1)$ for the previously defined polynomial f you can use the command

```
>>polyval(p, 1)
```

If you wanted to graph f for $0 \leq x \leq 5$ you could use the commands

```
x = linspace(0,5,100);
```

```
y = polyval(p, x);
```

```
plot(x, y)
```

To add two polynomials, just add the corresponding coefficient vectors. For example, to add the polynomials $f(x) = x^2 - 5x + 6$ and $g(x) = x^3 + 2x$ you could use the commands

```
p = [0 1 -5 6]; %Why do we need the 0 as the leading coefficient?
```

```
q = [1 0 2 0];
```

```
f_plus_g = p + q
```

To multiply two polynomials, use the `conv` command. For example, to multiply the polynomials $f(x) = x + 1$ and $g(x) = x - 1$ you could use the commands

```
p = [1 1];
q = [1 -1];
f_times_g = conv(p, q)
```

The command `deconv` divides two polynomials. For example, to carry out the division $\dfrac{x^3 + 2x}{x + 2}$ you could use the commands

```
num = [1 0 2 0];
den = [1 2];
[q, r] = deconv(num, den)
```

These commands produce the results `q = [1    -2    6]` and `r = [0    0    0    -12]`. This means $\dfrac{x^3 + 2x}{x + 2} = x^2 - 2x + 6 + \dfrac{-12}{x + 2}$

The remainder `r` will be zero if the numerator is divisible by the denominator. For example, try

```
num = [1 0 -1];
den = [1 1];
[q, r] = deconv(num, den)
```

To find the roots of a polynomial $f(x)$ (the values of x for which $f(x) = 0$), use the command `roots`. For example, to find the roots of the polynomials $f(x) = x^2 - 5x + 6$ and $g(x) = x^3 - x^2 + x - 1$ you could use the commands

```
p = [1 -5 6];
roots(p)
q = [1 -1 1 -1];
roots(q)
```

C. The Calculus of Polynomials

The command `polyder` can be used to find derivatives of polynomials, and `polyint` gives the integral. If you provide one input polynomial, the output of `polyder` will be the derivative of the input polynomial. If you provide two input polynomials, the output of `polyder` will either be the derivative of the product of the input polynomials (if you specify that there is just one output) or the derivative of the quotient of the input polynomials (if you specify that there are two outputs). For example, to find the derivatives of $f(x) = x^2 - 5x + 6$, $g(x) = (x - 1)(x + 1)$, and $h(x) = \dfrac{x - 1}{x + 1}$ you could use the commands

```
f_prime = polyder([1 -5 6])
g_prime = polyder([1 -1], [1 1])
[hpn, hpd] = polyder([1 -1], [1 1])
```

The last command produces the result `hpn = [2]` and `hpd = [1 2 1]`. This means $h'(x) = \dfrac{2}{x^2 + 2x + 1}$

D. Curve Fitting with Polynomials

A common problem in science and engineering is to find a function of a particular form that "best fits" a set of data points. Polynomial functions are often used for this purpose. The command `polyfit` generates a polynomial that best fits a given set of data points in the least-squares sense. You provide the coordinates of the data points and the degree of the polynomial, and `polyfit` provides the polynomial coefficients. For example, to find the line that best fits the 4 data points $(1, 2)$, $(3, 4)$, $(4, 6)$, and $(6, 5)$ you could use the commands

```
x = [1 3 4 6];

y = [2 4 6 5];

p1 = polyfit(x, y, 1) % The 1 means a polynomial of degree 1, i.e., a linear function

x1 = [0, 7];

plot(x, y, 'o', x1, polyval(p1, x1), '-b')

axis([0 7 0 7])
```

The slope m and y intercept b generated by `polyfit` minimize the sum of squares of the "errors:"

$$\sum_{i=1}^{n} [(mx_i + b) - y_i]^2$$

Practice Problems. (Problems 2 - 5 are from Gilat, *MATLAB: An Introduction with Applications*)

1. Define `x1 = 41:0.1:55` and `x2 = 0:100`. Calculate the mean and the standard deviation of the arrays `x1` and `x2`.

2. Plot the polynomial $y = -0.4x^4 + 7x^2 - 20.5x - 28$ in the domain $-5 \le x \le 4$. First create a vector of x values, next use the `polyval` function to calculate y, and then use the `plot` function.

3. Use MATLAB to carry out the following multiplication of two polynomials: $(2x^2 + 3)(x^3 + 3.5x^2 + 5x - 16)$.

4. Divide the polynomial $x^4 - 6x^3 + 13x^2 - 12x + 4$ by the polynomial $x^3 - 3x^2 + 2$.

5. The boiling temperature of water at various altitudes h is given in the following table. Determine a linear equation in the form $T_B = mh + b$ that best fits the data. Use the equation for calculating the boiling temperature at 16,000 ft. Make a plot of the points and the equation.

h (ft.)	0	2000	5000	7500	10000	20000	26000
T (°F)	212	210	203	198	194	178	168

1. For x1: 48 and 4.0847. For x2: 50 and 29.3002.

2.
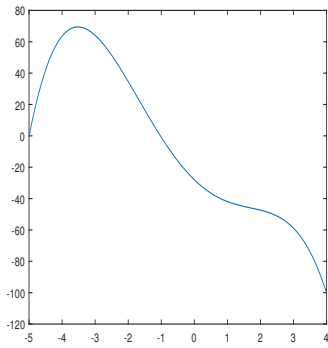
3. $2x^5 + 7x^4 + 13x^3 - 21.5x^2 + 15x - 48$

4. $x - 3 + \dfrac{4x^2 - 14x + 10}{x^3 - 3x^2 + 2}$

5. $T = -0.0017h + 211.8776.$ $T(16000) \approx 184.6$