MATH.2720 Introduction to Programming with MATLAB
Symbolic Utilities

A. Symbolic Math Using Mupad

Type the following command in the command window, which will open a new window called a mupad notebook:

```
>>mupad
```

A1. Algebra

1. MATLAB will factor polynomials. Type the following command in the mupad notebook next to the `[` symbol, then hit the Enter key.
   ```
   factor(x^4 - 5*x^2 + 4)
   ```

2. MATLAB can solve single equations. Try
   ```
   solve(x^4 - 5*x^2 + 4 = 0)
   ```

3. MATLAB can also solve systems of equations:
   ```
   solve({2*x + y = 5, x + 2*y = 4})
   ```

4. MATLAB can simplify expressions.
   Try these commands to simplify $\dfrac{x}{2x+1} + \dfrac{1}{x}$ and $\cos^3(x) + \cos(x)\sin^2(x)$
   ```
   simplify(x/(2*x + 1) + 1/x)
   simplify((cos(x))^3 + cos(x)*(sin(x))^2)
   ```

A2. Calculus

MATLAB can find limits, derivatives, and integrals symbolically. Try the following commands to find $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x}$, $\lim\limits_{x\to\infty} e^{-x}$, $\dfrac{d}{dx}\left[x^3\right]$, $\dfrac{d^2}{dx^2}\left[x^3\right]$, $\displaystyle\int \frac{1}{x^2+1}\,dx$, and $\displaystyle\int_0^1 \frac{x}{(x^2+1)^{3/2}}\,dx$, respectively.

```
limit(sin(x)/x, x=0)
limit(exp(-x), x=infinity)
diff(x^3, x)
diff(x^3, x$2)
int(1/(x^2+1), x)
int(x/(x^2+1)^(3/2), x=0..1)
```

A3. Differential Equations

MATLAB can even solve differential equations symbolically. Here are some examples:

1. To solve the d.e. $x^2y' + 2xy = 3x^2$, type the command
   `solve(ode(x^2*y'(x) + 2*x*y(x) = 3*x^2, y(x)))`
2. You can also solve initial value problems, such as $y' = y^2,\ y(1) = 1$:
   `solve(ode({y'(x) = y(x)^2, y(1) = 1}, y(x)))`
3. You can name the solution of an initial value problem, and you can even calculate the value of the solution at any x:
   `y1:=solve(ode({y''(x) - y(x) = 0, y(0) = 1, y'(0) = 2}, y(x)))`
   `float(y1 | x=2)`

B. Symbolic Math Using the Command Window

Here is how you can carry out the operations from the previous section using commands in the command window.

B1. Algebra

1. MATLAB will factor polynomials. Try the commands
   syms x y % This tells MATLAB to treat x and y as symbols rather than as arrays of numbers
   factor(x^4 - 5*x^2 + 4)

2. MATLAB can solve single equations. Try
   solve(x^4 - 5*x^2 + 4 == 0)

3. MATLAB can also solve systems of equations:
   [x, y] = solve(2*x + y == 5, x + 2*y == 4)

4. MATLAB can simplify expressions.
   Try these commands to simplify $\dfrac{z}{2z+1} + \dfrac{1}{z}$ and $\cos^3(x) + \cos(x)\sin^2(x)$
   clear x y
   syms x z
   simplify (z / (2*z + 1) + 1 / z)
   pretty(ans) % The "pretty" command makes the output easier to read.
   simplify ((cos(x))^3 + cos(x) * (sin(x))^2)

B2. Calculus

MATLAB can find limits, derivatives, and integrals symbolically. Try the following commands to find $\lim\limits_{x\to 0}\dfrac{\sin(x)}{x}$, $\lim\limits_{x\to\infty} e^{-x}$, $\dfrac{d}{dx}\left[x^3\right]$, $\dfrac{d^2}{dx^2}\left[x^3\right]$, $\displaystyle\int \frac{1}{x^2+1}\,dx$, and $\displaystyle\int_0^1 \frac{x}{(x^2+1)^{3/2}}\,dx$, respectively.

limit(sin(x)/x, x, 0)
limit(exp(-x), x, inf) % inf means infinity

```
diff(x^3)    % The diff command takes a derivative

diff(x^3, 2)    % The 2 means take the second derivative

int(1 / (x^2+1))  % The int command integrates

pretty( int(x / (x^2+1)^(3/2), 0, 1))
```

B3. Differential Equations

MATLAB can even solve differential equations symbolically. Here are some examples.

Type the commands

```
clear
syms x y(x)
```

1. To solve the d.e. $x^2y' + 2xy = 3x^2$, type the command
   `dsolve( x^2*diff(y) + 2*x*y == 3*x^2)`
2. You can also solve initial value problems, such as $y' = y(1-y),\ y(0) = 1/2$:
   `dsolve(diff(y)==y*(1 - y),  y(0)==1/2)`
3. Second derivatives are denoted `diff(y, 2)`, third derivatives `diff(y, 3)`, etc. To solve the second-order equation $y'' + y = 0$, type
   `dsolve(diff(y, 2) + y == 0)`
4. Of course, you can also add initial conditions to this problem, and you can give the solution a name. You can even calculate the value of the solution at any x value you like using the **subs** command.
   ```
   Dy = diff(y)
   y = dsolve(diff(y,2)+y==0, y(0)==1, Dy(0)==1)
   subs(y, pi/4)
   ```

Practice Problems

1. Factor the polynomial $x^3 - 3x^2 + 3x - 1$.
2. Find $\dfrac{d^2}{dx^2}\left[x\cosh(x)\right]$
3. Evaluate $\displaystyle\int_0^\infty e^{-x}\,dx$
4. Solve the initial value problem $y'' + 2y' + 5y = 20\cos(x),\ y(0) = 2,\ y'(0) = 0$

Answers to Practice Problems

1. $(x-1)^3$ 2. $2\sinh(x) + x\cosh(x)$ 3. 1
4. $4\cos(x) + 2\sin(x) - 2e^{-x}\left(2\cos(x)^2 - 1\right) - 4e^{-x}\cos(x)\sin(x)$
or $4\cos(x) + 2\sin(x) - 2e^{-x}\cos(2x) - 2e^{-x}\sin(2x)$