MATH.2720 Introduction to Programming with MATLAB
Symbolic Utilities

Click on the "New Live Script" button on the toolbar. This will open a Live Editor window.

Algebra

1. MATLAB will factor polynomials. Enter the following commands in the Live Editor window, then click the Run key.

```
syms x y
factor(x^4 - 5*x^2 + 4)
```

2. MATLAB can solve single equations. Enter the following command in the Live Editor window, then click the Run key.

```
solve( {x^4 - 5*x^2 + 4 == 0} ) %Note the double equal sign
```

3. MATLAB can also solve systems of equations:

```
[x, y] = solve({2*x + y == 5, x + 2*y == 4})
```

4. MATLAB can simplify expressions.

   Try these commands to simplify $\dfrac{x}{2x+1} + \dfrac{1}{x}$ and $\cos^3(x) + \cos(x)\sin^2(x)$

```
clear
syms x
simplify(x/(2*x + 1) + 1/x)
simplify((cos(x))^3 + cos(x)*(sin(x))^2)
```

Calculus

MATLAB can find limits, derivatives, and integrals symbolically. Try the following commands to find $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x}$, $\lim\limits_{x\to\infty} e^{-x}$, $\dfrac{d}{dx}\left[x^3\right]$, $\dfrac{d^2}{dx^2}\left[x^3\right]$, $\displaystyle\int \frac{1}{x^2+1}\,dx$, and $\displaystyle\int_0^1 \frac{x}{(x^2+1)^{3/2}}\,dx$, respectively.

```
limit(sin(x)/x, x, 0)
limit(exp(-x), x, inf) %inf means infinity
diff(x^3, x)
diff(x^3, x, 2) %The 2 means find the second derivative
int(1/(x^2+1), x)
int(x/(x^2+1)^(3/2), 0, 1)
```

OVER

Differential Equations

MATLAB can even solve differential equations symbolically. Here are some examples:

1. To solve the d.e. $x^2y' + 2xy = 3x^2$, use the commands

```
clear
syms y(x)
dsolve( x^2 * diff(y) + 2*x*y(x) == 3*x^2)
```

2. You can also solve initial value problems, such as $y' = y(1-y),\ y(1) = 1/2$:

```
dsolve(diff(y)==y(x)*(1-y(x)),y(0)==1/2)
```

3. Here is how to solve the second order d.e. $y'' = y$:

```
Dy = diff(y);
dsolve(diff(Dy)==y(x))
```

4. You can name the solution of an initial value problem, and you can even calculate the value of the solution at any x:

```
Dy = diff(y);
y = dsolve(diff(Dy)+y(x)==0,y(0)==1,Dy(0)==1)
subs(y,pi/4) %This evaluates the solution y at x = pi/4
```

Practice Problems

1. Factor the polynomial $x^3 - 3x^2 + 3x - 1$.
2. Find $\dfrac{d^2}{dx^2}\left[x\cosh(x)\right]$ (MATLAB knows the hyperbolic functions.)
3. Evaluate $\displaystyle\int_0^\infty e^{-x}\,dx$
4. Solve the initial value problem $y'' + 2y' + 5y = 20\cos(x),\ y(0) = 2,\ y'(0) = 0$

Answers to Practice Problems

1. $(x-1)^3$ 2. $2\sinh(x) + x\cosh(x)$ 3. 1
4. $4\cos(x) + 2\sin(x) - 2e^{-x}\left(2\cos(x)^2 - 1\right) - 4e^{-x}\cos(x)\sin(x)$
or $4\cos(x) + 2\sin(x) - 2e^{-x}\cos(2x) - 2e^{-x}\sin(2x)$