MATH.2720 Introduction to Programming with MATLAB
Miscellaneous Useful Utilities

## A. Roots of Nonlinear Equations

We have already seen that the `roots` function can be used to find the roots of a polynomial. For example, to solve the equation $x^3+2x^2+3x+4 = 0$ you can use the command `roots([1, 2, 3, 4])`.

To find a root of a function other than a polynomial, you can use the function `fzero`. The syntax is either

`fzero(@f, x0)` or `fzero(@f, [x1, x2])`

Here `f` is the name of the function file defining the function whose root you want, `x0` is an initial estimate of the root, and `[x1, x2]` is an interval with the property that `f(x1)` and `f(x2)` have opposite signs. If you use the command `fzero(@f, [x1, x2])` MATLAB will look for a root in the interval `[x1, x2]`.

For example, if you want to find a root of the function given by $f(x) = \cos(x) - x$ near $x = 0$ you can first create a function file named `f.m` containing the lines

```
function y = f(x)
y = cos(x)-x;
end
```

and then issue the command `[x, fval] = fzero(@f, 0)` in the command window. The output `x` is the value of the root, and the output `fval` is the value of $f(x)$.

If you don't know what value to use as your initial estimate, you can always graph the function and see approximately where the graph crosses the horizontal axis.

The `fzero` utility only works on functions of a single variable, but sometimes you may want to find a root of a function that depends on a parameter as well as a variable. For example, suppose you want to find a root of the function given by $f(x) = k\cos(x) - x$ where $k$ is a constant. First create a function file named `f.m` containing the lines

```
function y = f(x, k)
y = k*cos(x)-x;
end
```

If you want to find a root of $f$ when $k = 2$, issue the command `fzero(@(x)f(x,2), 0)` in the command window.

## B. Optimization

In many applications, you need to maximize or minimize a function. The MATLAB function `fminbnd` attempts to minimize a function of one variable. The syntax is

`[x, fval] = fminbnd(@f, x1, x2)`

Here `f` is the name of the function file defining the function you want to minimize, and `[x1, x2]` is the interval on which you want to minimize `f`. The output `x` is the point at which `f` achieves its minimum value, and `fval` is the value of `f` at that point.

For example, if you want to minimize the function given by $f(x) = xe^{-x^2}$ over the interval $[-2, 2]$ you can first create a function file named `f.m` containing the lines

```
function y = f(x)
y = x.*exp(-x.^2);
end
```

and then issue the command `[x, fval] = fminbnd(@f, -2, 2)` in the command window.

## C. Numerical Integration (for those of you who have taken Calculus II)

Not every function has an antiderivative that can be expressed in terms of elementary functions. In such cases, you may need to approximate a definite integral numerically. The MATLAB function `quad` calculates an approximate values of a definite integral. The syntax is

```
quad(@f, a, b)
```

Here `f` is the name of the function file defining the function you want to integrate, and `[a, b]` is the interval of integration. For example, to approximate the value of $\int_0^1 e^{-x^2}\,dx$ you can first create a function file named `f.m` containing the lines

```
function y = f(x)
y = exp(-x.^2);
end
```

and then issue the command `quad(@f, 0, 1)` in the command window.

## D. Eigenvalues and Eigenvectors (for those of you who have studied linear algebra)

The function `eig` generates the eigenvalues and eigenvectors of a square matrix A. The syntax is

```
[X, D] = eig(A)
```

The eigenvalues are the diagonal entries of $D$, and the corresponding (normalized) eigenvectors are the corresponding columns of $X$. Try

```
A = [6  -2  -1;-2  6  -1;-1  -1  5]
[X, D] = eig(A)
```

## E. Reference

1. Higham and Higham, *MATLAB Guide*, 2nd ed., SIAM, 2005.

## F. Practice Problems

1. Find a root of the function given by $f(x) = x^2 - e^x$ on the interval $[-2, 2]$.

2. Minimze the function given by $f(x) = \dfrac{e^x}{x^2}$ over the interval $[0.5, 3]$.

3. Estimate the value of the integral $\int_0^1 \sin\left(\sqrt{x}\right)\,dx$

4. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} -1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & -1 \end{pmatrix}$$

Partial Answers to Practice Problems

1. $-0.70347$ 2. $x \approx 2$ 3. $0.60233$ 4. Eigenvalues are $-4$, $-2$, and $2$.