MATH.2720 Introduction to Programming with MATLAB   Spring 2017

Final Exam

Due 5PM May 5

This is an exam, so the work you submit must be your own.

Thanks to Dr. Colby and Dr. Gamache for the information they provided.

Please email me (`stephen_pennell@uml.edu`) your function files `regula_falsi.m`, `mixing_ratio.m`, and `Delta_q.m`; your script file `wet_bulb.m`; and your answers to part 2 and the last question in part 5.

Part 1. (40 pts.) Write a function file called `regula_falsi.m` that implements the Regula Falsi Method described in Appendix A and discussed in class.

- There should be 4 inputs to `regula_falsi.m`: the name of the (continuous) function whose root you are looking for, 2 numbers x_1 and x_2 (with $x_1 < x_2$), and the error tolerance. The values of the function at x_1 and x_2 should have opposite signs, guaranteeing that the interval $[x_1, x_2]$ contains a root of the function.
- There should be 1 output from `regula_falsi.m`: an estimate r of the value of a root of the given function. The value of the function at r should be no greater than the error tolerance in absolute value.
- `regula_falsi.m` should check to make sure that the values of the function at x_1 and x_2 have opposite signs. If this is not the case, the output of `regula_falsi.m` should be `NaN`.
- Be sure to include comments describing the function and its inputs and output.

Part 2. (5 pts.) Test your function `regula_falsi.m` on the function $f(x) = x^2 - 2$ on the interval $[0, 2]$ with tolerance $\epsilon = 0.001$.

Part 3. (15 pts.) Write a function file called `mixing_ratio.m` that calculates the mixing ratio for air as described in Appendix B.

- There should be 2 inputs to `mixing_ratio.m`: temperature T and pressure p.
- There should be 1 output from `mixing_ratio.m`: the value of the mixing ratio w for the given values of T and p.
- Be sure to include comments describing the function and its inputs and output.

Part 4. (15 pts.) Write a function file called `Delta_q.m` that calculates the value of the function $\Delta q\,(T)$ defined in Appendix C.

- There should be 1 input to `Delta_q.m`: the value of T.
- There should be 1 output from `Delta_q.m`: the value of $\Delta q\,(T)$.
- Declare the parameters T_a, T_d, and p_a to be `global`.
- Be sure to include comments describing the function and its inputs and output.

Part 5. (25 pts.) Write a script file called `wet_bulb.m` that calculates wetbulb temperature by using your function `regula_falsi.m` to find the root of $\Delta q\,(T)$. Use T_d and T_a as the endpoints of the interval containing the root. Use tolerance $\epsilon = 0.001$.

- Declare the parameters T_a, T_d, and p_a to be `global`.
- Ask the user to input the air temperature T_a (in °F), the dew point T_d (in °F), and the atmospheric pressure p_a (in hectopascals).
- Convert T_a and T_d to degrees K.
- Display the calculated value of the wetbulb temperature in °F.
- Be sure to include comments describing the script file.
- Test your code using $T_a = 79.5$°F, $T_d = 55.2$°F, and $p_a = 1013.6$ hPa. (UMass Lowell data from July 10, 2008, provided by Dr. Colby.) You should get a value of 64.1°F.

Appendix A - The Regula Falsi Method

The Regula Falsi Method is an iterative method that produces a sequence of increasingly accurate approximations to the root of a function.

Input: A continuous function f, an interval $[x_1, x_2]$ with the property that $f(x_1)$ and $f(x_2)$ have opposite signs, and the error tolerance ϵ. (The condition that $f(x_1)$ and $f(x_2)$ have opposite signs guarantees that f has at least one root in the interval $[x_1, x_2]$.)

Output: An approximate value of a root of f in the interval $[x_1, x_2]$.

How the method works: At each step, define $x_3 = \dfrac{f(x_2)x_1 - f(x_1)x_2}{f(x_2) - f(x_1)}$. If $|f(x_3)| < \epsilon$ you are finished, and x_3 is the estimate of the root. Otherwise, set $x_2 = x_3$ if $f(x_1)$ and $f(x_3)$ have opposite signs, or set $x_1 = x_3$ if $f(x_1)$ and $f(x_3)$ have the same sign. The new interval $[x_1, x_2]$ contains a root of f and is smaller than the old $[x_1, x_2]$. Repeat these steps until you find a value of x_3 for which $|f(x_3)| < \epsilon$.

Example: We will estimate the value of $\sqrt{2}$ by using the bisection search method to find a root of the function given by $f(x) = x^2 - 2$ on the interval $[0, 2]$ using tolerance $\epsilon = 0.001$. Note that $f(0)f(2) = (-2)(2) < 0$, so $f(0)$ and $f(2)$ have opposite signs.

Iteration 1. The interval $[x_1, x_2] = [0, 2]$ is known to contain a root of f.
$x_3 = \dfrac{f(2)(0) - f(0)(2)}{f(2) - f(0)} = \dfrac{(2)(0) - (-2)(2)}{2 - (-2)} = 1$. $|f(1)| = |-1| = 1 \geq \epsilon$ so we continue. $f(x_1)$ and $f(x_3)$ have the same sign (both negative), so we redefine x_1 to have the value 1 and leave x_2 as 2.

Iteration 2. The interval $[x_1, x_2] = [1, 2]$ is known to contain a root of f.
$x_3 = \dfrac{f(2)(1) - f(1)(2)}{f(2) - f(1)} = \dfrac{(2)(1) - (-1)(2)}{2 - (-1)} = 4/3$. $|f(4/3)| = |-2/9| = 2/9 \geq \epsilon$ so we continue. $f(x_1)$ and $f(x_3)$ have the same sign (both negative), so we redefine x_1 to have the value $4/3$ and leave x_2 as 2.

Iteration 3. The interval $[x_1, x_2] = [4/3, 2]$ is known to contain a root of f.
$x_3 = \dfrac{f(2)(4/3) - f(4/3)(2)}{f(2) - f(4/3)} = \dfrac{(2)(4/3) - (-2/9)(2)}{2 - (-2/9)} = 7/5$. $|f(7/5)| = |-1/25| = 1/25 \geq \epsilon$ so we continue. $f(x_1)$ and $f(x_3)$ have the same sign (both negative), so we redefine x_1 to have the value $7/5$ and leave x_2 as 2.

This process continues until $|f(x_3)|$ drops below ϵ.

Appendix B - Mixing Ratio

The mixing ratio w is the ratio of the mass of water vapor to the mass of dry air. The mixing ratio corresponding to air temperature T and air pressure p is calculated as follows:

$$w = \frac{0.62197\ p_v}{p - p_v}$$

where p_v denotes vapor pressure. The vapor pressure is calculated in terms of air temperature T as follows:

$$p_v = 10^b$$

where

$$\begin{aligned}
p_0 &= 1013.246 \\
T_0 &= 373.16 \\
a_1 &= 11.344\,(1 - T/T_0) \\
a_2 &= -3.49149\,(T_0/T - 1) \\
b_1 &= -7.90298\,(T_0/T - 1) \\
b_2 &= 5.02808\log_{10}(T_0/T) \\
b_3 &= -1.3816\,(10^{a_1} - 1)/10^7 \\
b_4 &= 8.1328\,(10^{a_2} - 1)/10^3 \\
b_5 &= \log_{10}(p_0) \text{ and} \\
b &= b_1 + b_2 + b_3 + b_4 + b_5
\end{aligned}$$

Appendix C - Wetbulb Temperature

As explained by Dr. Colby, wetbulb temperature is the temperature a parcel of air can be cooled to by evaporating water into it at constant pressure. Given the air temperature T_a (K), the dew point T_d (K), and the atmospheric pressure p_a (hectopascals), the wetbulb temperature can be calculated by finding the root of the function Δq given by

$$\Delta q\,(T) = \frac{L\,(w_2 - w_1)}{1 + w_2} - c_p\,(T_a - T)\,(1 + 0.8w_2)$$

where $c_p = 1005$ J/(kg K) is the heat capacity of water vapor; $L = 2.501 \times 10^6$ J/kg is the latent heat of water vapor at 0°C; w_1 denotes the mixing ratio corresponding to temperature T_d and pressure p_a, and w_2 denotes the mixing ratio corresponding to temperature T and pressure p_a. **Use your function `mixing_ratio` to calculate w_1 and w_2.** The units of T are degrees K.