MATH.2720 Introduction to Programming with MATLAB Spring 2019

Final Exam

Due 11:59:59PM May 6

This is an exam, so the work you submit must be your own. Do not talk to anyone other than me about this exam.

Thanks to Dr. Colby and Dr. Gamache for the information they provided.

Please email me (`stephen_pennell@uml.edu`) your function files `coord_convert.m`, `mixing_ratio.m`, and `Delta_q.m` as well as your script file `wet_bulb.m`

Part I. Coordinate Conversion (25 pts.)

Write a function file called `coord_convert.m` that converts 3D rectangular coordinates to spherical coordinates or vice versa.

- There should be 4 inputs to `coord_convert.m`: coordinates `a`, `b`, and `c` and a character string called `coord` that has the value `'rectangular'` if the input coordinates are rectangular coordinates or `'spherical'` if the input coordinates are spherical coordinates.
- There should be 3 outputs from `coord_convert.m`: coordinates `u`, `v`, and `w` in the new coordinate system. The values of `u`, `v`, and `w` should be calculated as described in Appendix A.
- If `coord` is anything other than `'rectangular'` or `'spherical'` the values of the outputs `u`, `v`, and `w` should all be assigned the value `NaN`
- Be sure to include comments describing the function and its inputs and output.

If you want to test your code, you can use the fact that the point with rectangular coordinates $(1, 1, 0)$ has spherical coordinates $\left(\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4}\right)$.

Part II. Calculating Wet Bulb Temperature (75 pts.)

II.1. (20 pts.) Write a function file called `mixing_ratio.m` that calculates the mixing ratio for air as described in Appendix B.

- There should be 2 inputs to `mixing_ratio.m`: temperature T (K) and pressure p (hectopascals).
- There should be 1 output from `mixing_ratio.m`: the value of the mixing ratio w for the given values of T and p.
- Be sure to include comments describing the function and its inputs and output.

If you want to test your code, `mixing_ratio(300, 1000)` ≈ 0.0228.

II.2. (20 pts.) Write a function file called `Delta_q.m` that calculates the value of the function Δq defined in Appendix C.

- There should be 4 inputs to `Delta_q.m`: the values of T (K), atmospheric temperature T_a (K), dew point T_d (K), and atmospheric pressure p_a (hectopascals).
- There should be 1 output from `Delta_q.m`: the value of $\Delta q\,(T, T_a, T_d, p_a)$ (J/kg).
- Be sure to include comments describing the function and its inputs and output.

If you want to test your code, `Delta_q(290, 300, 280, 1000)` $\approx$ `4.5193e+03`

II.3. (35 pts.) Write a script file called `wet_bulb.m` that calculates wetbulb temperature by using the MATLAB utility `fzero` to find the root of the function Δq. Use T_d and T_a as the endpoints of the interval containing the root. Because `fzero` can only find the root of a function of one variable, you will have to use an anonymous function as input to `fzero`, as in the second example on the handout on Miscellaneous Useful Utilities.

- Ask the user to input the air temperature T_a (in °F), the dew point T_d (in °F), and the atmospheric pressure p_a (in hectopascals).
- Convert the units of T_a and T_d to K.
- Use `fzero` to find the root of $\Delta q\,(T, T_a, T_d, p_a)$ for the given values of T_a, T_d, and p_a.
- Display the calculated value of the wetbulb temperature in °F.
- Be sure to include comments describing the script file.
- Test your code using $T_a = 79.5$°F, $T_d = 55.2$°F, and $p_a = 1013.6$ hPa. (UMass Lowell data from July 10, 2008, provided by Dr. Colby.) You should get a value of 64.1°F.

Appendix A - Coordinate Conversion Formulas

If the input coordinates a, b, and c are rectangular coordinates, calculate the corresponding spherical coordinates as follows.

$$u = \sqrt{a^2 + b^2 + c^2}$$

$$v = \begin{cases} \cos^{-1}\left(c/\sqrt{a^2+b^2+c^2}\right) & \text{if } a^2+b^2+c^2 > 0 \\ \texttt{NaN} & \text{if } a^2+b^2+c^2 = 0 \end{cases}$$

$$w = \begin{cases} \tan^{-1}(b/a) & \text{if } a > 0 \\ \pi + \tan^{-1}(b/a) & \text{if } a < 0 \\ \pi/2 & \text{if } a = 0 \text{ and } b > 0 \\ -\pi/2 & \text{if } a = 0 \text{ and } b < 0 \\ \texttt{NaN} & \text{if } a = 0 \text{ and } b = 0 \end{cases}$$

If the input coordinates a, b, and c are spherical coordinates, calculate the corresponding rectangular coordinates as follows.

$$u = \begin{cases} a\sin(b)\cos(c) & \text{if } a \geq 0 \\ \texttt{NaN} & \text{if } a < 0 \end{cases}$$

$$v = \begin{cases} a\sin(b)\sin(c) & \text{if } a \geq 0 \\ \texttt{NaN} & \text{if } a < 0 \end{cases}$$

$$w = \begin{cases} a\cos(b) & \text{if } a \geq 0 \\ \texttt{NaN} & \text{if } a < 0 \end{cases}$$

Appendix B - Mixing Ratio

The mixing ratio w is the ratio of the mass of water vapor to the mass of dry air. The mixing ratio corresponding to air temperature T and air pressure p is calculated as follows:

$$w = \frac{0.62197 \; p_v}{p - p_v}$$

where p_v denotes vapor pressure. The vapor pressure is calculated in terms of air temperature T as follows:

$$p_v = 10^b$$

where

$$\begin{aligned}
p_0 &= 1013.246 \\
T_0 &= 373.16 \\
a_1 &= 11.344 \left(1 - \frac{T}{T_0}\right) \\
a_2 &= 3.49149 \left(1 - \frac{T_0}{T}\right) \\
b_1 &= 7.90298 \left(1 - \frac{T_0}{T}\right) \\
b_2 &= 5.02808 \log_{10} \left(T_0/T\right) \\
b_3 &= 1.3816 \left(1 - 10^{a_1}\right)/10^7 \\
b_4 &= 8.1328 \left(10^{a_2} - 1\right)/10^3 \\
b_5 &= \log_{10}(p_0) \text{ and} \\
b &= b_1 + b_2 + b_3 + b_4 + b_5
\end{aligned}$$

Appendix C - Wetbulb Temperature

As explained by Dr. Colby, wetbulb temperature is the temperature a parcel of air can be cooled to by evaporating water into it at constant pressure. Given the air temperature T_a (K), the dew point T_d (K), and the atmospheric pressure p_a (hectopascals), the wetbulb temperature can be calculated by finding the value of T for which $\Delta q\left(T, T_a, T_d, p_a\right) = 0$, where

$$\Delta q\left(T, T_a, T_d, p_a\right) = \frac{L\left(w_2 - w_1\right)}{1 + w_2} - c_p\left(T_a - T\right)\left(1 + 0.8 w_2\right)$$

Here $c_p = 1005$ J/(kg K) is the heat capacity of water vapor; $L = 2.501 \times 10^6$ J/kg is the latent heat of water vapor at 0°C; w_1 denotes the mixing ratio corresponding to temperature T_d and pressure p_a, and w_2 denotes the mixing ratio corresponding to temperature T and pressure p_a. **Use your function `mixing_ratio` to calculate w_1 and w_2.** The units of T are degrees K.