Bonferroni Method for considering the overall confidence level of several confidence intervals taken together -- handout from Shelley Rasmussen

We have to be careful when we calculate multiple confidence intervals (or do multiple tests of hypotheses) because confidence (or significance) levels may not be what we think they are.

Suppose, for example, that we have four independent random samples, one from each of four populations with unknown means Mu1, Mu2, Mu3 and Mu4, respectively. We plan to calculate 95% confidence intervals for each of Mu1 through Mu4. Before we do the experiment and/or collect the data, we guess that the probability that a population mean is in the calculated confidence interval is about 0.95. Since the samples in this case are independent, so are the intervals. So we estimate that the probability that each of the four population means will be in its respective confidence interval is:

$$0.95 \times 0.95 \times 0.95 \times 0.95 = 0.81$$

We say the confidence level of the four intervals taken together is about 81%, much lower than the confidence level for an individual interval.

This should make sense since in everyday life we would expect to have a better chance of making one correct guess than of making four correct guesses in a row.

The **Bonferroni Method** allows us to estimate the overall confidence level of several confidence intervals taken together (whether they come from independent samples or not). Suppose we have k intervals, each of confidence level A (for example, A = 0.95 if we want individual 95% confidence intervals). Then the confidence level of the k intervals taken together is greater than or equal to:

$$1 - k \times (1 - A)$$

In the example above, A = 0.95 and k = 4, so we would use the Bonferroni Method to estimate that the overall confidence level of the four intervals taken together is at least

$$1 - 4 \times (1 - 0.95) = 1 - 4 \times (0.05) = 0.80$$

This value of 0.80 agrees well with the 0.81 we found above, when we assumed independence.

If we had let each of the four intervals in the example be 99% confidence intervals, then the overall confidence level of the intervals taken together would be greater than or equal to 0.96.

As this example shows, we have to make the individual confidence intervals wider (increase the individual confidence levels) in order to control the overall confidence level.