1. (5pts) The switch in the circuit has been open for a long time before closed at $t = 0$. Find $v(t)$ for $t \geq 0$.

$V(0) = 12 \, \text{V}$

$V(\infty) = \frac{12 \times 7}{12 + 6} \times 12 = \frac{12 \times 2}{3} = 8 \, \text{V}$

$R_{th} = 12 \frac{1}{12 + 4} = 12 \frac{1}{16} = 0.75 \, \Omega$

$\frac{1}{RC} = \frac{1}{0.5 \times 4} = \frac{1}{2}$

$V(t) = V(\infty) + (V(0) - V(\infty)) e^{-\frac{1}{RC} t}$

$= 8 + 4 e^{-\frac{1}{2} t} \, \text{V}$
2. (5pts). Find \( v(t) \) for \( t > 0 \). (You may use superposition to find \( v(\infty) \)).

For \( t < 0 \)

By current division
\[
\tilde{i}_2 = \frac{15}{15 + 30 + 45} \times 3 = 0.9 \text{A}
\]
\[
U_2 = 30 \tilde{i}_2 = 30 \times 0.9 = 27 \text{V}
\]
\[
U_1 = 5 \times 3 = 15 \text{V}
\]
\[
V(0) = U_1 + U_2 = 15 + 27 = 42 \text{V}
\]

For \( t > 0 \)

Both 3A and 30V are on.

\( U(\infty) \) due to 3A = \( U(0) = 42 \text{V} \)

\( V_6(\infty) \) due to 30V

By voltage division
\[
V_6(\infty) = \frac{30}{30 + 15 + 5} \times 30 = \frac{900}{50} = 18 \text{V}
\]
\[
U(\infty) = U_6(\infty) + U_b(\infty) = 42 + 18 = 60 \text{V}
\]

\[
R_{th} = 5 + 3 + \frac{30}{11} (15 + 5) = 5 + 3 + 12 = 20 \Omega
\]

\[
\frac{1}{RC} = \frac{1}{20 \times 10 \times 10^{-3}} = 5
\]

\[
v(t) = 60 + (42 - 60) e^{-5t} = 60 - 18 e^{-5t} \text{V}
\]
3. (5pts) The switch has been open for a long time before it is closed at $t = 0$. Find $i(t)$ and $i_x(t)$ for $t \geq 0$.

For $t > 0$

$$\frac{R_{th}}{L} = \frac{6}{2} = 3$$

$$i(t) = 3.5 + (1.5 - 3.5) e^{-3t} = 3.5 - 2e^{-3t} \text{ A}$$

As $t \to \infty$

$$\frac{\dot{i}_x}{4} = \frac{u_2 + v_i}{4} = \frac{L}{4} \frac{di}{dt} + 4 \dot{i}_x$$

$$\dot{i}_x(t) = \frac{2}{4} \frac{di}{dt} + 4 \dot{i}_x$$

$$= 2 \times 6 e^{-3t} + 14 - 8 e^{-3t}$$

$$= 3.5 + e^{-3t} \text{ A}$$
4. (5 pts) The switch has been closed for a long time before it is open at $t = 0$. Find $v(t)$ and $v_R(t)$ for $t > 0$.

By current division

$$\frac{\nu(0)}{12 + 6 \times 3} = 2A$$

$$\nu(0) = \frac{12}{12 + 6} \times 3 = 2A$$

$$V(0) = \frac{6 + 2}{2} = 12V$$

For $t > 0$

$$v(\infty) = \frac{3 \times 12}{2} = 36V$$

$$\nu(0^+) = \dot{\nu}(0) = 2A$$

$$\frac{dv(0^+)}{dt} = \frac{1}{c} \nu(0^+) = 16 \times 2 = 32V/s$$

$$R = 12\Omega$$

$$\alpha = \frac{R}{2L} = \frac{12}{2 \times 2} = 3$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{8}, \quad \alpha > \omega_0, \text{ Case I}$$

$$s_1, s_2 = -3 \pm \sqrt{9-1} = -2, -4$$

$$v(t) = 36 + A_1 e^{-2t} + A_2 e^{-4t}$$

$$\frac{dv}{dt} = -2A_1 e^{-2t} - 4A_2 e^{-4t}$$

$$v(0) = 36 + A_1 + A_2 = 12$$

$$\nu(0) = -2A_1 - 4A_2 = 32$$

$$v(t) = 36 - 32 e^{-2t} + 8 e^{-4t}$$

$$v_R(t) = 12 \frac{dv}{dt} = 12 \left(3 - \frac{1}{16} (64 e^{-2t} - 32 e^{-4t})\right)$$

$$= 12 \left(3 - 4 e^{-2t} + 2 e^{-4t}\right)$$
5. (5pts) The switch has been closed for a long time before open at \( t = 0 \). Find \( v(t) \) for \( t > 0 \).

\[ A+ t = 0^+ \quad \frac{dv(0^+)}{dt} = \frac{1}{C} \cdot \frac{\dot{v}_L(0)}{2} = 100 \text{ V/s} \]

For \( t > 0 \) \[ R_{th} = 2.4 \frac{1}{(12 + 12)} = 12 \Omega \]

By current division
\[ \dot{v}_1 = \frac{12}{12 + 12 + 24} \times 4 = 1 \text{ A} \]
\[ v(\infty) = 24 \dot{v}_1 = 24 \text{ V} \]

\[ \alpha = \frac{R_{th}}{2L} = \frac{12}{2} = 6, \quad \omega_0 = \frac{1}{\sqrt{L C}} = 10 \]
\[ \alpha < \omega_0, \quad \text{Case 3,} \quad C \omega_0^2 - \alpha^2 = \sqrt{100 - 36} = 8 \]
\[ V(t) = 24 + e^{-6t}(B_1 \cos 8t + B_2 \sin 8t) \]
\[ V(0) = 24 + B_1 = 0 \quad B_1 = -24 \]
\[ \dot{V}(0) = -6B_1 + 8B_2 = 100 \quad B_2 = \frac{-100 + 8 \times 24}{8} = -5.5 \]
\[ V(t) = 24 + e^{-6t}(-24 \cos 8t - 5.5 \sin 8t) \]