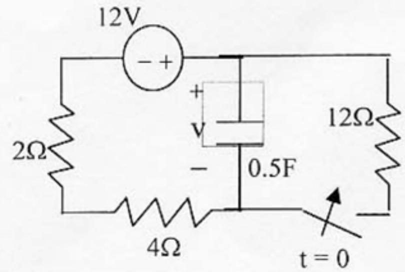
Name:________________________________

Circuit Theory I (16.201): **Final Exam Practice A solution**

Total 25 pts

1. (5pts) The switch in the circuit has been open for a long time before closed at t = 0.
   Find $v(t)$ for $t \geq 0$.



$v(0) = 12\ V$

$v(\infty) = \dfrac{12}{12+6} \times 12 = \dfrac{12}{18} \times 12 = 8V$

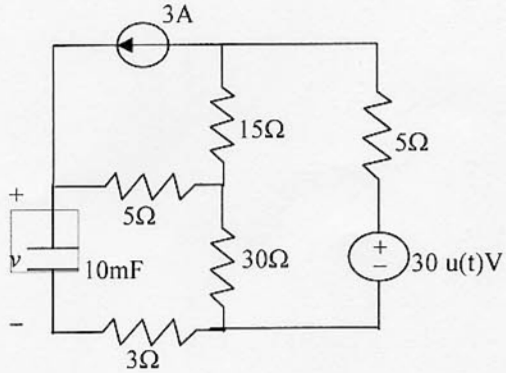
$R_{th} = 12 \,//\, (2+4) = 12 // 6 = 4\Omega$

$\dfrac{1}{RC} = \dfrac{1}{0.5 \times 4} = \dfrac{1}{2}$

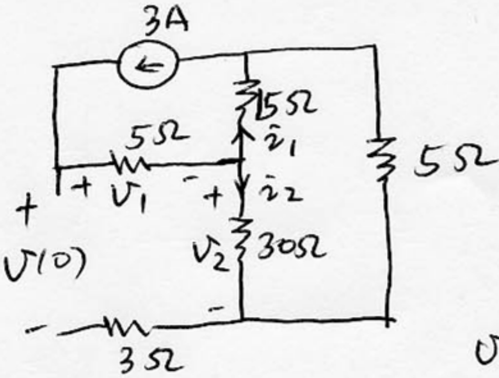
$$v(t) = v(\infty) + (v(0) - v(\infty))\, e^{-\frac{1}{RC}t}$$

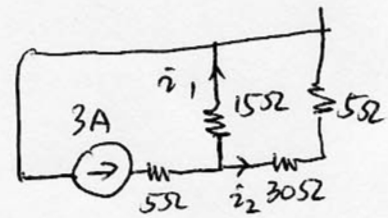
$$= 8 + 4e^{-\frac{1}{2}t} \quad V$$

2. (5pts). Find v(t) for t > 0. (You may use superposition to find $v(\infty)$).



For $t<0$





By current division

$$i_2 = \frac{15}{15+30+5}\times 3 = 0.9A$$

$$v_2 = 30 i_2 = 30\times 0.9 = 27V$$

$$v_1 = 5\times 3 = 15V$$

$$v(0) = v_1 + v_2$$

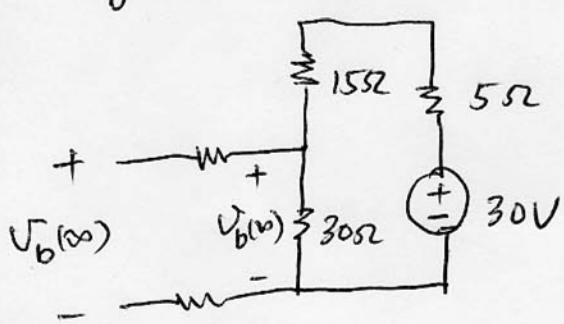
$$v(0) = v_1 + v_2 = 15 + 27 = 42V$$

For $t>0$

Both 3A and 30V are on.

$v_a(\infty)$ due to 3A $= v(0) = 42V$

$v_b(\infty)$ due to 30V



By voltage division

$$v_b(\infty) = \frac{30}{30+15+5}\times 30 = \frac{900}{50} = 18V$$

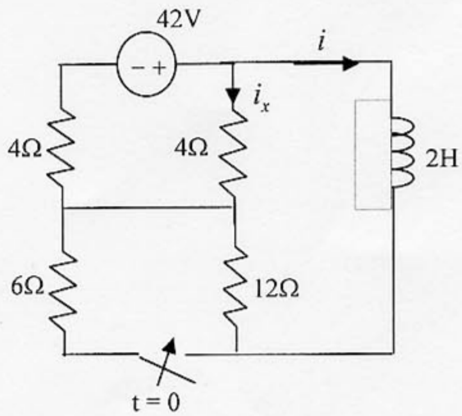
$$v(\infty) = v_a(\infty) + v_b(\infty) = 42 + 18 = 60V$$

$$R_{th} = 5 + 3 + 30\,//\,(15+5) = 5 + 3 + 12 = 20\Omega$$

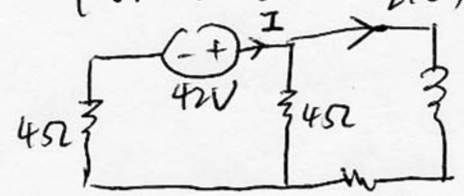
$$\frac{1}{RC} = \frac{1}{20\times 10\times 10^{-3}} = 5$$

$$v(t) = 60 + (42-60)e^{-5t} = 60 - 18e^{-5t} \text{ V}$$

3. (5pts) The switch has been open for a long time before it is closed at t = 0. Find i(t) and i_x(t) for t ≥ 0.



For $t<0$



R_{eq} w.r.t 42V

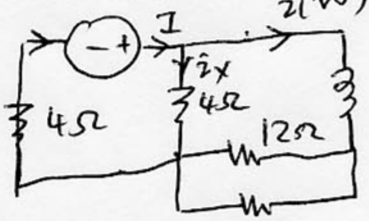
$R_{eq} = 4 + 4 // 12 = 7\Omega$

$I = \frac{42}{R_{th}} = 6A$

By current division

$i(0) = \frac{4}{4+12} \times 6 = 1.5A$

For $t > 0$



R_{eq} w.r.t 42V

$R_{eq} = 4 + 4 // 12 // 6 = 6\Omega$

$I = \frac{42}{R_{th}} = 7A$

By current division

$i(\infty) = \frac{4}{4+4} \times 7 = 3.5A$

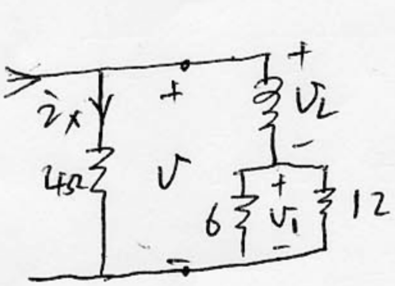
R_{th} w.r.t inductor

$R_{th} = 4 + 4 // 4 = 6\Omega$

$\frac{R_{th}}{L} = \frac{6}{2} = 3$

$i(t) = 3.5 + (1.5 - 3.5)e^{-3t} = 3.5 - 2e^{-3t}$ A

As to i_x



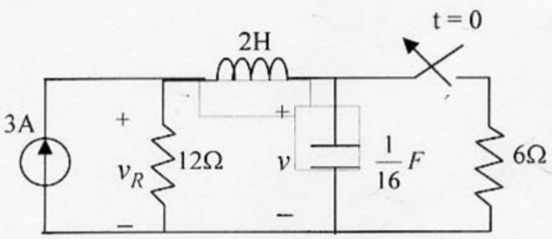
$i_x = \frac{v(t)}{4} = \frac{v_L + v_1}{4} = \frac{L\frac{di}{dt} + 4i}{4}$

$i_x(t) = \frac{2\frac{di}{dt} + 4i}{4}$

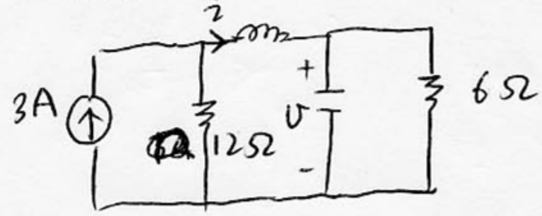
$= \frac{2 \times 6e^{-3t} + 14 - 8e^{-3t}}{4}$

$= 3.5 + e^{-3t}$ A

4. (5pts) The switch has been closed for a long time before it is open at t = 0. Find v(t) and v_R(t) for t > 0.



At $t = 0^-$

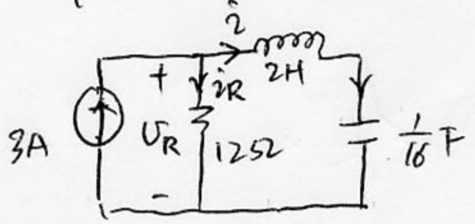


By current division

$i(0) = \frac{12}{12+6} \times 3 = 2A$

$v(0) = 6 \times 2 = 12V$

For $t > 0$



$v(\infty) = 3 \times 12 = 36V$

$i(0^+) = i(0) = 2A$

$\frac{dv(0^+)}{dt} = \frac{1}{C} i(0^+) = 16 \times 2 = 32 V/s$

$R = 12\Omega$, $\quad \alpha = \frac{R}{2L} = \frac{12}{2 \times 2} = 3$

$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{8}$, $\quad \alpha > \omega_0$, Case 1

$s_1, s_2 = -3 \pm \sqrt{9-1} = -2, -4$

$v(t) = 36 + A_1 e^{-2t} + A_2 e^{-4t}$ V

$\frac{dv}{dt} = -2A_1 e^{-2t} - 4A_2 e^{-4t}$

$v(0) = 36 + A_1 + A_2 = 12 \quad \Rightarrow \quad 72 + 2A_1 + 2A_2 = 24$

$\dot{v}(0) = -2A_1 - 4A_2 = 32 \qquad\qquad -2A_1 - 4A_2 = 32 \quad +$

$72 - 2A_2 = 56$, $\quad A_2 = 8$

$A_1 = \frac{-4 \times 8 - 32}{2} = -32$

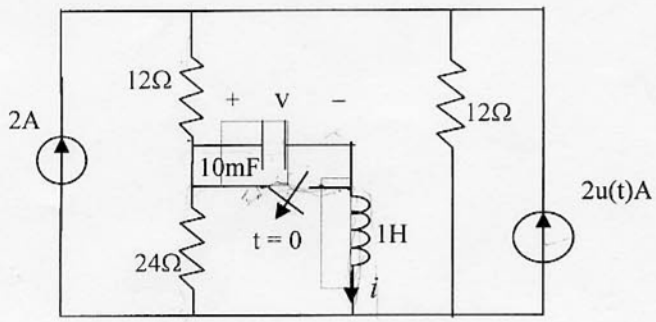
$v(t) = 36 - 32e^{-2t} + 8e^{-4t}$ V

$v_R(t) = 12\, i_R = 12(3 - i) = 12(3 - C\frac{dv}{dt})$

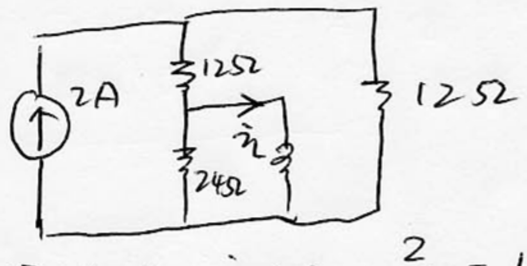
$= 12(3 - \frac{1}{16}(64e^{-2t} - 32e^{-4t}))$

$= 12(3 - 4e^{-2t} + 2e^{-4t})$ V

5. (5pts) The switch has been closed for a long time before open at t = 0. Find $v(t)$ for t > 0.



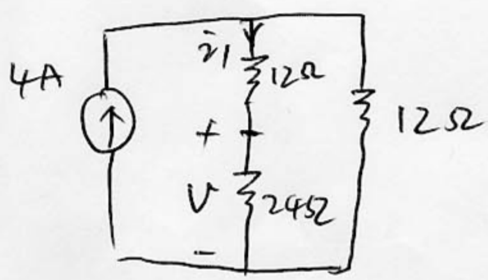
At $t = 0^-$ $2u(t) = 0$



$\underline{v_c(0) = 0,}\quad i_L(0) = \frac{2}{2} = 1A$

At $t = 0^+$ $\frac{dv(0^+)}{dt} = \frac{1}{C} i_L(0) = \underline{100\ V/s}$

For $t > 0$ $R_{th} = 24 \,||\, (12 + 12) = 12\Omega$



By current division

$i_1 = \frac{12}{12+12+24} \times 4 = 1A$

$\underline{v(\infty) = 24 i_1 = 24V}$

$\alpha = \frac{R_{th}}{2L} = \frac{12}{2} = 6, \quad \omega_0 = \frac{1}{\sqrt{LC}} = 10$

$\alpha < \omega_0$, Case 3, $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{100 - 36} = 8$

$v(t) = 24 + e^{-6t}(B_1 \cos 8t + B_2 \sin 8t)$

$v(0) = 24 + B_1 = 0$ $B_1 = -24$

$\dot{v}(0) = -6B_1 + 8B_2 = 100$ $B_2 = \frac{+100 - 6 \times 24}{8} = -5.5$

$v(t) = 24 + e^{-6t}(-24 \cos 8t - 5.5 \sin 8t)$ ✓