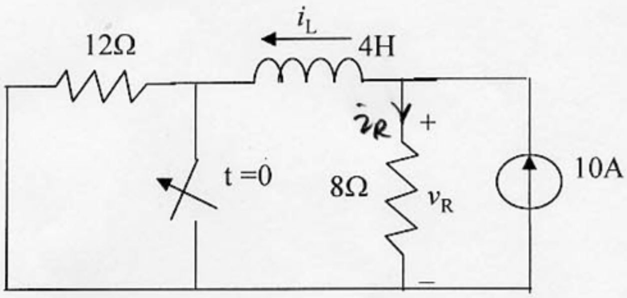
Name:______________________________

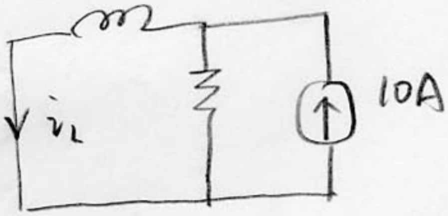
Circuit Theory I (16.201): **Final Exam Practice B solution**

Total 25 pts

1. (5) The switch in the circuit has been closed for a long time before opening at t = 0. Find $i_L(t)$, and $v_R(t)$ for $t \geq 0$.

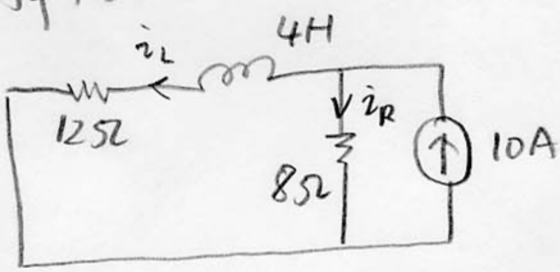


Before switch, $t<0$



$i_L(0) = 10A$

After switch $t>0$, By current division



$$i_L(\infty) = \frac{8}{8+12} \times 10 = 4A$$

R_{th} w r t

$$R_{th} = 12 + 8 = 20\,\Omega$$

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-\frac{R_{th}}{L}t}$$

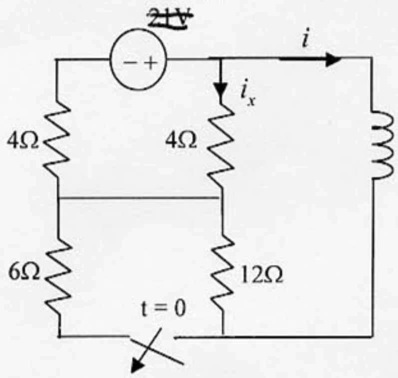
$$= 4 + (10-4)\,e^{-\frac{20}{4}t}$$

$$= 4 + 6e^{-5t}\ A$$

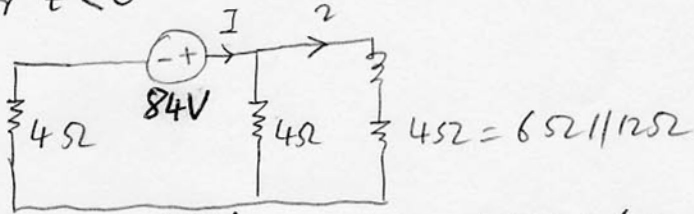
$$v_R(t) = 8i_R = 8(10 - i_L(t)) = 8(6 - 6e^{-5t})$$

$$v_R(t) = 48(1 - e^{-5t})\ V$$

2. (5pts) The switch has been closed for a long time before it is open at t = 0. Find $i(t)$ and $i_x(t)$ for $t \geq 0$.



For $t<0$

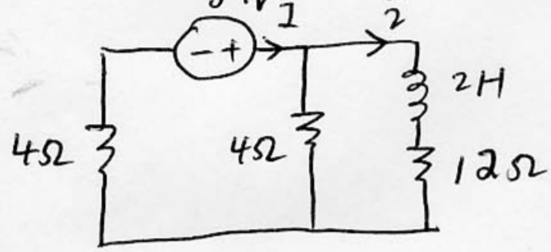


84V

R_{eq} w.r.t $\quad R_{eq} = 4 + 4 // 4 = 6\Omega$

$I = \frac{84}{6} = 14A, \quad i = \frac{14}{2} = 7A, \quad \underline{i(0) = 7A}$

For $t>0$



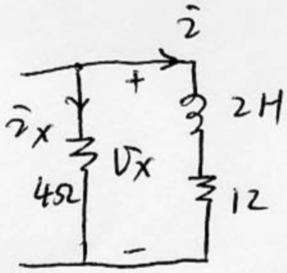
Under DC Condition

$$I = \frac{84}{4 + 4//12} = \frac{84}{7} = 12A$$

$$\underline{i(\infty)} = \frac{4}{4+12} \times 12 = \frac{4}{16} \times 12 = \underline{3A}$$

R_{th} w.r.t $\quad R_{th} = 12 + 4 // 4 = 14\Omega$

$$i(t) = 3 + (7-3)e^{-\frac{14}{2}t} = 3 + 4e^{-7t} \text{ A}$$



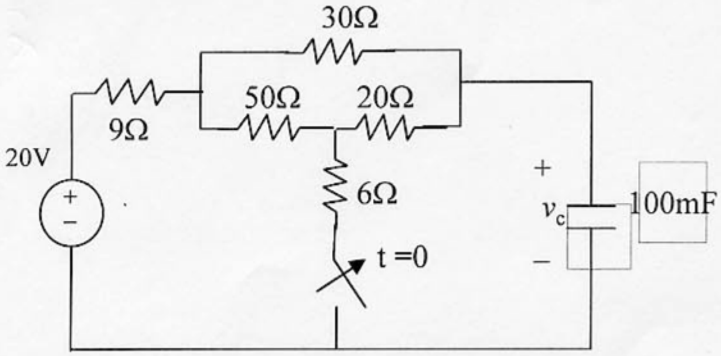
$$v_x = L\frac{di}{dt} + 12i$$

$$i_x = \frac{v_x}{4} = \frac{2}{4}\frac{di}{dt} + \frac{12}{4}i$$

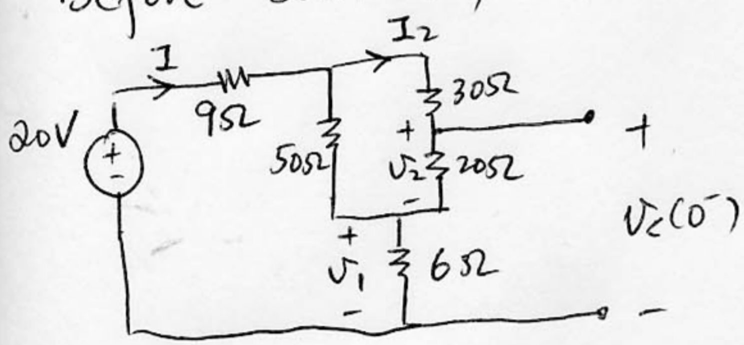
$$i_x(t) = \frac{1}{2}\frac{di}{dt} + 3i = -14e^{-7t} + 9 + 12e^{-7t}$$

$$= 9 - 2e^{-7t} \text{ A}$$

3. The switch has been closed for a long time before open at t = 0. Find $v_c(t)$ for t > 0.



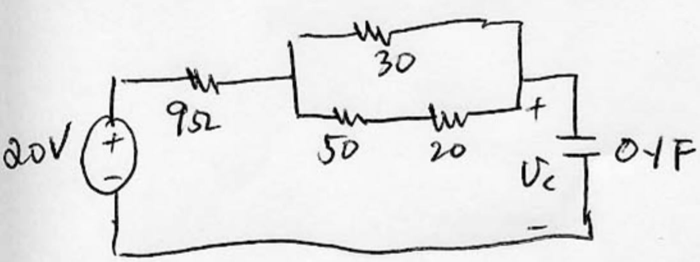
Before switch, at $t=0^-$



R_{eq} w.r.t 20V

$$R_{eq} = 9 + 50 \| (30+20) + 6\Omega$$
$$= 9 + 25 + 6 = 40\Omega.$$
$$I = \frac{20}{R_{eq}} = 0.5A, \quad I_2 = 0.5I = 0.25A$$
$$v_1 = 6I = 6 \times 0.5 = 3V,$$
$$v_2 = 20 I_2 = 20 \times 0.25 = 5V$$
$$v_c(0) = v_1 + v_2 = 3 + 5 = 8V$$

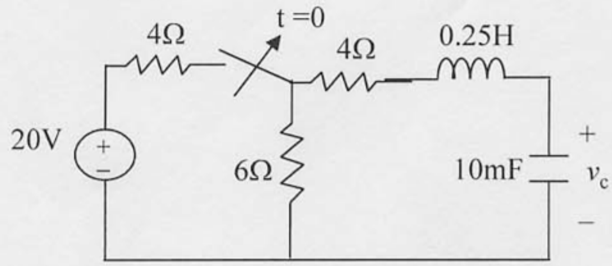
After switch



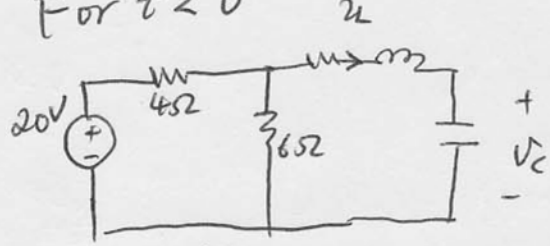
$$R_{th} = 9 + 30 \| (50+20)$$
$$= 9 + \frac{30 \times 70}{100} = 30\Omega$$
$$v_c(\infty) = 20V, \quad R_{th}C = 30 \times 0.1 = 3$$

$$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty)) e^{-\frac{1}{R_{th}C}t}$$
$$= 20 + (8-20) e^{-\frac{1}{3}t}$$
$$= 20 - 12 e^{-\frac{1}{3}t} \text{ V.}$$

4. (5pts) For t <0, the switch is closed. Assume that a steady state has been reached by t = 0. At t = 0, the switch is open. Find $v_c(t)$ for t>0.



For $t<0$



$$V_c(0) = \frac{6}{6+4} \times 20 = 12V, \quad i_L(0) = 0$$

For $t>0$, R_{th} w.r.t —ᗰᗰ—||— $\quad R_{th} = 4+6 = 10\Omega$

At $t=0^+$ $\quad i_c(0^+) = i_L(0^+) = 0, \quad \frac{dV_c(0)}{dt} = \frac{1}{C} i_c(0^+) = 0$

$$\alpha = \frac{R_{th}}{2L} = \frac{10}{2\times 0.25} = 20, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25\times 0.01}} = 20$$

$\alpha = \omega_0$, $\Rightarrow$ case 2.

Since $V_c(\infty) = 0$, the general solution is

$$V_c(t) = (A_1 + A_2 t) e^{-20t}$$

$$\dot{V}_c(t) = (A_1 + A_2 t)(-20) e^{-20t} + A_2 e^{-20t}$$

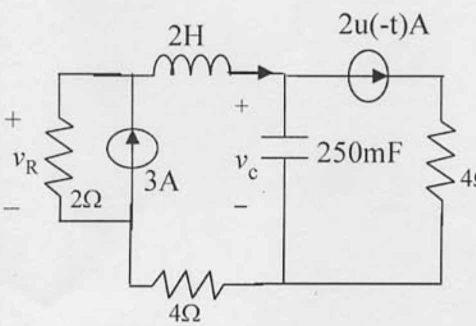
By initial condition, $\quad V_c(0) = A_1 = 12$

$$\dot{V}_c(0) = -20 A_1 + A_2 = 0.$$

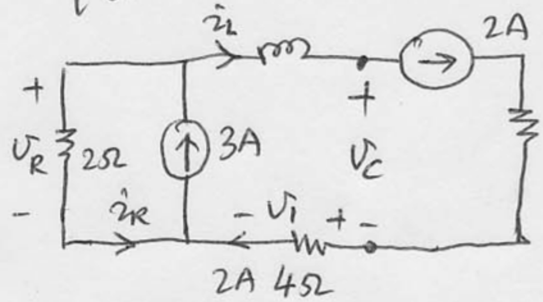
$\Rightarrow \quad A_1 = 12, \quad A_2 = 240.$

Finally $\quad V_c(t) = (12 + 240t) e^{-20t} \; V$

5. (5pts) Find $v_c(t)$ and $v_R(t)$ for $t > 0$. Recall that $u(-t)=1$ for $t<0$ and $u(-t)=0$ for $t>0$.



For $t < 0$ $\quad 2u(-t) = 2A$



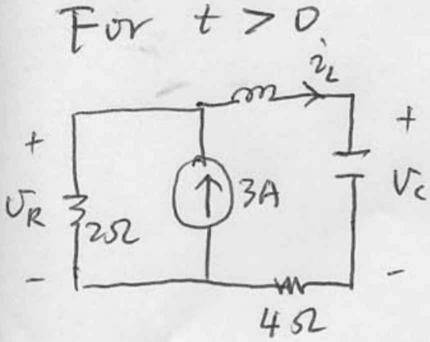
$i_L(0) = 2A$

$V_1 = 4 \times 2 = 8V$

By KCL, $i_R = 3-2 = 1A$, $V_R = 2V$

By KVL, $V_c = V_R - V_1 = 2 - 8 = -6V$, $\underline{V_c(0) = -6V}$

For $t > 0$



At $t = 0^+$, $i_c(0^+) = i_L(0^+) = 2A$.

$$\frac{dV_c(0^+)}{dt} = \frac{1}{C} i_L(0^+) = \frac{1}{0.25} \times 2 = \underline{8V/s}$$

At $t = \infty$, $\underline{V_c(\infty) = 3 \times 2 = 6V}$.

$R_{th} = 4 + 2 = \underline{6\Omega}$, $\quad \alpha = \frac{R_{th}}{2L} = \frac{6}{2 \times 2} = 1.5$

$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{2}$

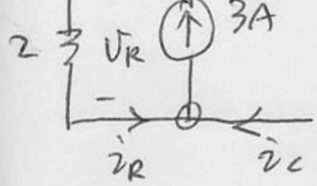
$\alpha > \omega_0$, case 1.

$s_1, s_2 = -1.5 \pm \sqrt{1.5^2 - 2} = -1.5 \pm 0.5$,

$s_1 = -1$, $\quad s_2 = -2$, $\quad V_c(t) = 6 + A_1 e^{-t} + A_2 e^{-2t}$

By initial Condition, $\quad V_c(0) = 6 + A_1 + A_2 = -6$
$\dot{V}_c(0) = -A_1 - 2A_2 = 8$ $\Rightarrow \begin{cases} A_1 = -16 \\ A_2 = 4 \end{cases}$

$V_c(t) = 6 - 16e^{-t} + 4e^{-2t}$ V



$V_R = 2i_R = 2(3 - i_c) = 2(3 - C\frac{dV_c}{dt})$

$V_R(t) = 2(3 - 0.25 \times (16e^{-t} - 8e^{-2t}))$

$= 6 - 8e^{-t} + 4e^{-2t}$ V