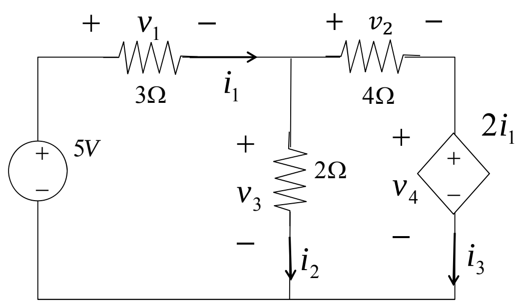
Chapter 2 Basic Laws

A typical circuit in Chapters 2, 3:



We need three basic Laws:

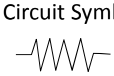
- Ohm's law
- Kirchhoff's current law (KCL)
- Kirchhoff's voltage law (KVL)

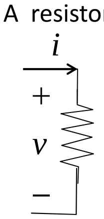
Chapter 2 will use these laws to solve simple circuits

Chapter 3, 4, develop systematic methods to solve complex circuits.

We will be asked to find some of the variables:
v_1, v_2, v_3, v_4, i_1, i_2, i_3.

§ 2.2 Ohm's Law – Property of a resistor

	Circuit Symbol	Notation
A resistor:		R



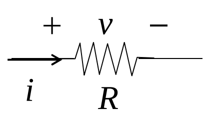
Suppose voltage v and current i are assigned according to passive convention

Ohm's Law: Voltage across a resistor is directly proportional to the current flowing through it: $v \propto i$, $\Rightarrow$ v / i = constant.

Resistance: is defined as the ratio v/i, i.e.,

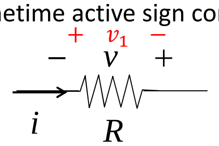
$\mathrm{R} \triangleq \frac{v}{i}$ Measured in ohm (Ω) 1Ω = 1V/A

For a resistor



$v = Ri$, ⇔ $i = \frac{v}{R}$

Sometime active sign convention cannot be avoided:



$v = -Ri$, $i = -\frac{v}{R}$

$v_1 = -v$
$v_1 = Ri$
$v = -v_1 = -Ri$

Resistance depends on the material and structure

For a material with cross-sectional area A and length l,

$$R = \rho \frac{l}{A}$$

* ρ is the resistivity, measured in Ωm

Resistivity for some conductors:

Material	ρ (Ω•m) at 20 °C
Silver	1.59×10^{-8}
Copper	1.68×10^{-8}
Gold	2.44×10^{-8}
Aluminium	2.82×10^{-8}
Calcium	3.36×10^{-8}
Zinc	5.90×10^{-8}
Nickel	6.99×10^{-8}
Lithium	9.28×10^{-8}
Iron	1.0×10^{-7}

Semi-conductors:

Material	ρ (Ω•m) at 20 °C
Carbon	4×10^{-5} to 8×10^{-4}
Germanium	4.6×10^{-1}
Sea water	2×10^{-1}
Drinking water	2×10^{1} to 2×10^{3}
Silicon	6.40×10^{2}

Insulators:

Material	ρ (Ω•m) at 20 °C
Glass	10×10^{10} to 10×10^{14}
Hard rubber	1×10^{13}
Sulfur	1×10^{15}
Air	1.3×10^{16} to 3.3×10^{16}
Teflon	10×10^{22} to 10×10^{24}

L3

Two special resistors:

- R= 0, v = Ri=0.
  No matter how much current flows in it, no voltage drop → a short circuit.
  A wire made of conductor can be considered as a short circuit.
- R= ∞, i = v/R =0.
  No current flows in it for any voltage. → an open circuit.
  Usually two disconnected points, or two points separated by insulator.

Types of resistors:

- Fixed resistor,
- Variable resistor
- Linear resistor
- Nonlinear resistor

We only consider fixed linear resistors.

Conductance: The reciprocal of resistance

$$G \triangleq \frac{1}{R} = \frac{i}{v}$$

Measured in mho, or Siemens (S)

Conductance measures the ability to conduct electric current.
With the same voltage applied, larger conductance yields larger current.

Power consumed by a resistor:

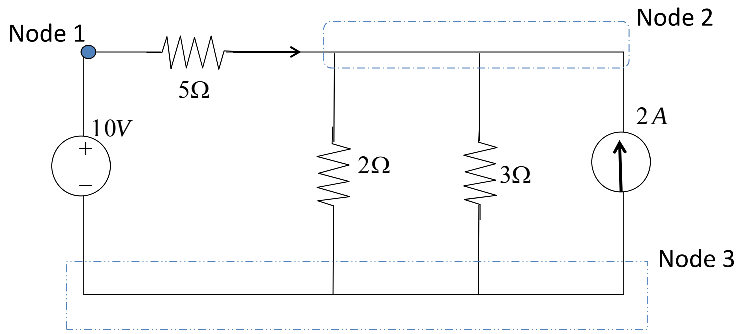
$$p = vi = Ri \times i = Ri^2$$

$$p = vi = v\frac{v}{R} = \frac{v^2}{R}$$

$p \geq 0$
Always consumes power

L3

§ 2.3 Nodes, branches, loops

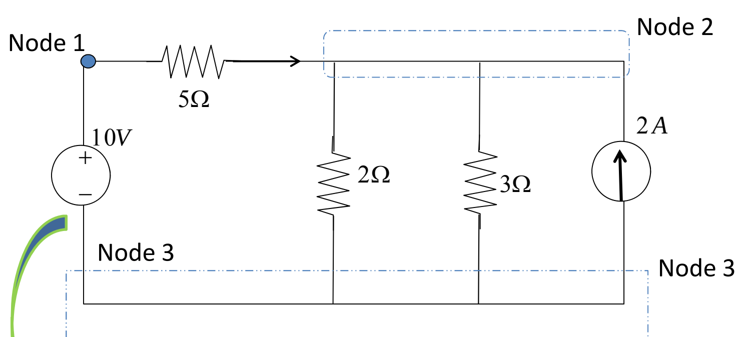


<u>Branch:</u> represents a single element, a source or a resistor. For the above circuit, there are 5 branches.
<u>Node:</u> a point connecting two or more branches, or a group of points, a piece of wire, where there is no voltage drop.

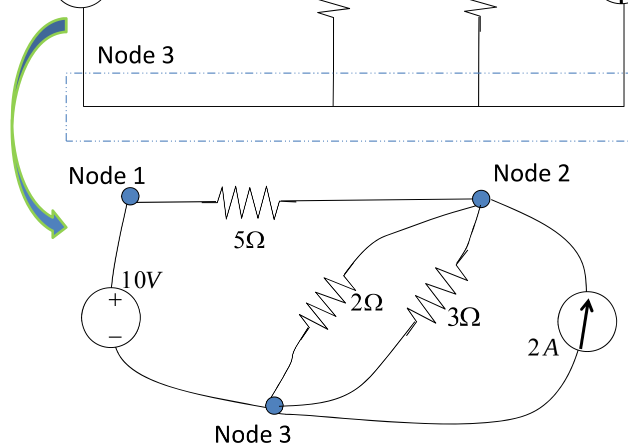
The above circuit has only 3 nodes.

L3

An equivalent circuit can be drawn to clearly show the nodes



A piece of wire can be replaced with a single point.



Loop: any closed path.

It is hard to count all loops. In chapter 3, we use mesh, a simple loop that does not contain any other loop in it.

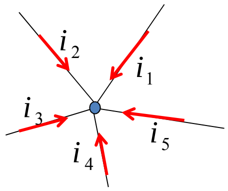
§ 2.4 Kirchhoff's Laws

KCL: Kirchhoff's current law
KVL: Kirchhoff's voltage law.

Both established in 1847

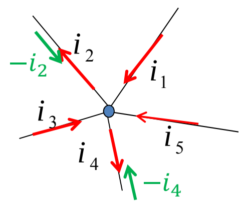
KCL is based on the law of conservation of charge.

<u>KCL Law</u>:
the algebraic sum of currents entering a node (or a closed boundary) is 0.



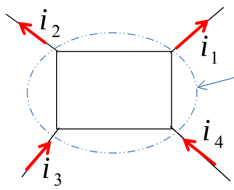
$$i_1 + i_2 + i_3 + i_4 + i_5 = 0$$

" +" for entering
" − " for leaving



$$i_1 - i_2 + i_3 - i_4 + i_5 = 0$$

$$i_1 + i_3 + i_5 = i_2 + i_4$$

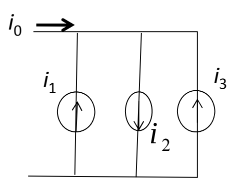


Closed boundary

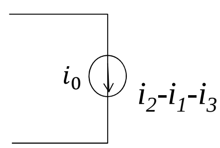
$$-i_1 - i_2 + i_3 + i_4 = 0$$

Sum of currents leaving
= sum of currents entering

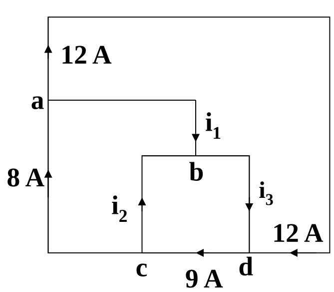
Application: use KCL to combine currents.



$i_0 + i_1 - i_2 + i_3 = 0,$
$\Rightarrow\ i_0 = i_2 - i_1 - i_3$



Example: Use KCL to find i_1, i_2, i_3



KCL at node a:
$8 - 12 - i_1 = 0$
$\Rightarrow i_1 = -4A$

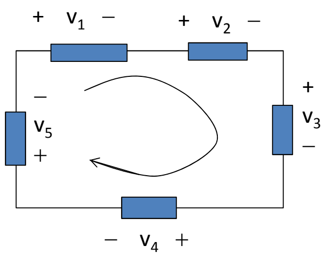
At node c:
$9 - i_2 - 8 = 0,$
$\Rightarrow i_2 = 1A$

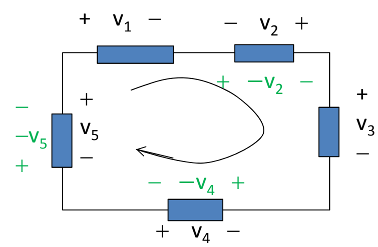
At node d:
$i_3 + 12 - 9 = 0,$
$\Rightarrow i_3 = -3A$

L3

KVL is based on the law of conservation of energy.

KVL law:
The algebraic sum of all voltage drops around a loop, along clockwise (or counter clockwise) direction, is zero.





Along clockwise direction:
From + to – : voltage drop
From – to + : voltage rise
All are voltage drop.
By KVL,
$v_1+v_2+v_3+v_4+v_5= 0$

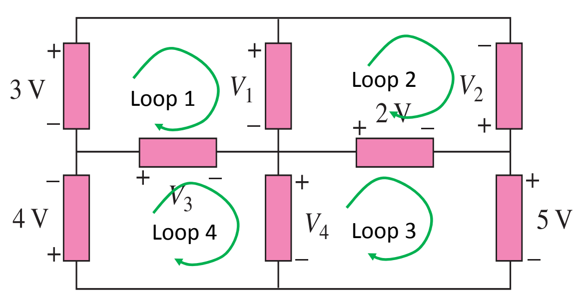
Voltage drop: v_1, v_3
Voltage rise: v_2,v_4,v_5

By KVL, $v_1-v_2+v_3-v_4-v_5= 0$

For voltage drop, use "+"
For voltage rise, use "-"

L3

Example:

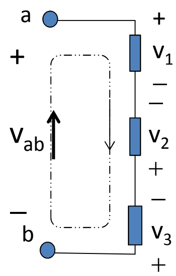


KVL along loop 3
$2+5-v_4=0$, $v_4=7V$
Loop 4:
$v_3+v_4+4=0$, $v_3=-11V$
Loop 1:
$v_1-v_3-3=0$, $v_1=-8V$
Loop 2:
$-v_2-2-v_1=0$, $v_2=6V$.

L3

Use KVL to add up voltages:

Given v_1, v_2, v_3, what is v_{ab}?



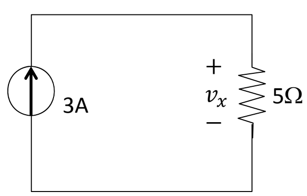
Make a loop to include the open circuit from b to a.
KVL along clockwise direction,

$v_1 - v_2 - v_3 - v_{ab} = 0, \Rightarrow \quad v_{ab} = v_1 - v_2 - v_3$

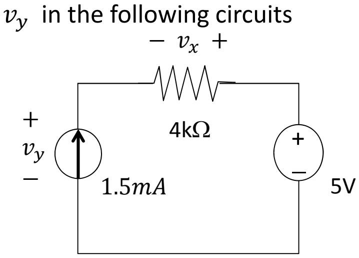
The voltage drop from a to b is the sum of voltage drops along a path from a to b.

R3

Practice problem 1: Find v_x, v_y in the following circuits

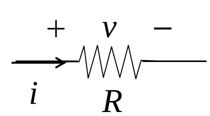


(a)

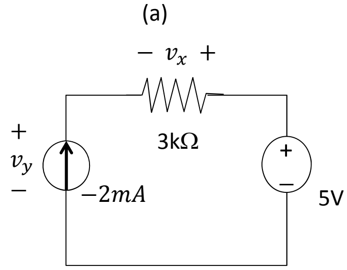


(b)

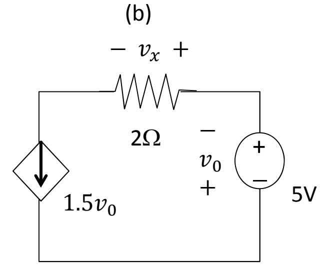
Ohm's Law:



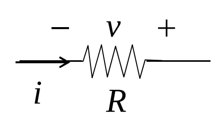
$v = Ri,$



(c)

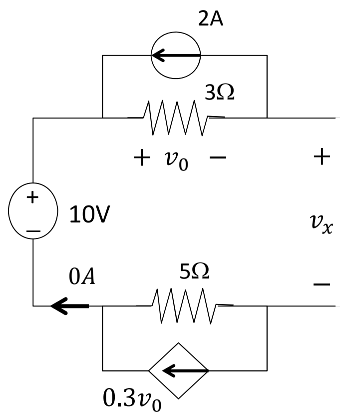


(d)



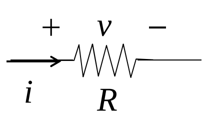
$v = -Ri$

Practice 2: Find v_x

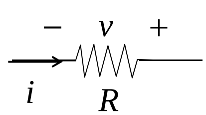


Basic Laws

Ohm's Law:

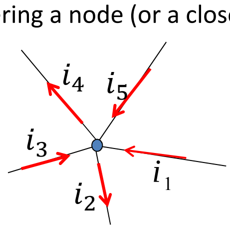


$v = Ri,$



$v = -Ri$

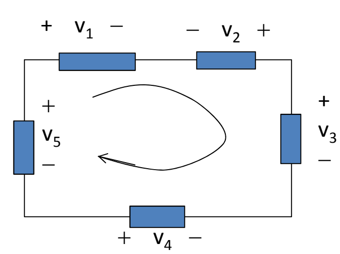
KCL Law: the algebraic sum of currents entering a node (or a closed boundary) is 0.



$$i_1 - i_2 + i_3 - i_4 + i_5 = 0$$

" +" for entering
" − " for leaving

KVL law: the algebraic sum of all voltage drops around a loop, along clockwise (or counter clockwise) direction, is zero.

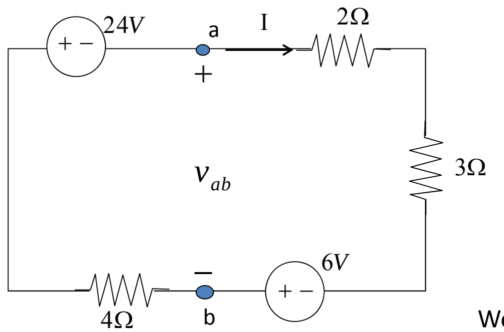


By KVL, $v_1 - v_2 + v_3 - v_4 - v_5 = 0$

For voltage drop, use "+"
For voltage rise, use "-"

Use basic laws to solve simple circuit problems.

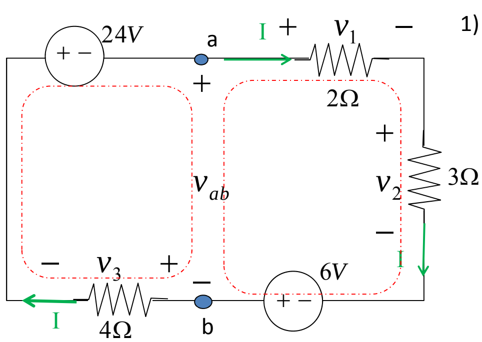
Example: Find current I and v_{ab} .



Main idea:
Transform the circuit problem into a math problem: using all three basic laws to make an equation for current I.

We need to assign auxiliary variables to make each step clear.
We assign resistor voltages, v_1, v_2, v_3.

Solution:



1) By KCL, same current flows through all elements in clockwise direction.

2) Voltage for each element is either given, or, can be expressed in terms of the current I, by Ohm's law.

For convenience, the voltages v_1,v_2,v_3 are assigned according to passive sign convention. Then,

$$v_1=2I,\ v_2=3I,\ v_3=4I \qquad \text{(E1)}$$

3) Use KVL to form an equation:

$$24 + v_1 + v_2 - 6 + v_3 = 0 \qquad \text{(E2)}$$

4) Plug (E1) into (E2) to obtain equation for I

$24 + 2I + 3I - 6 + 4I = 0,\ \Rightarrow\ 9I= -18,\ \underline{I= - 2A}$

How to find v_{ab}?

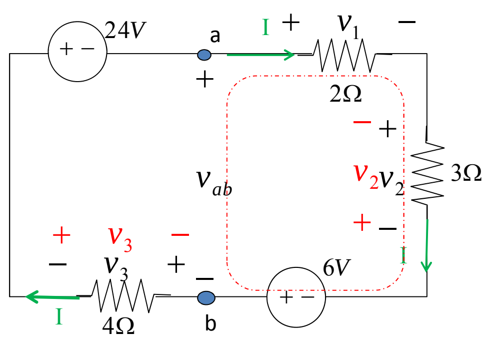
5) For v_{ab}, by KVL, $v_1 + v_2 - 6 - v_{ab} = 0$.
Since $v_1=2I=-4V$, $v_2=3I= -6V$,
we obtain $v_{ab}= -6+v_1+v_2=-6-4-6= \underline{-16V}$

Alternatively, work on left side loop

$$24 + v_{ab} + v_3 = 0$$
$$v_{ab} = -24 - 4I = -16V$$

L4

What if some resistor voltages are assigned the opposite way?



Let us reverse v_2, v_3.

$v_1 = 2I$, $v_2 = -3I$, $v_3 = -4I$ (E1)

3) Use KVL to form an equation:

$24 + v_1 - v_2 - 6 - v_3 = 0$ (E2)

4) Plug (E1) into (E2) to obtain equation for I

$24 + 2I - (-3I) - 6 - (-4I) = 0,$

Same equation for I:

$\Rightarrow 9I = -18, \; I = -2A$

5) For v_{ab}, by KVL, $v_1 - v_2 - 6 - v_{ab} = 0.$

Since $v_1 = 2I = -4V$, $v_2 = -3I = 6V$,

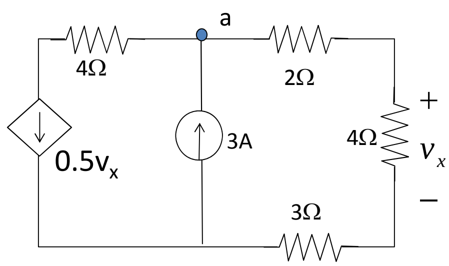
we obtain $v_{ab} = -6 + v_1 - v_2 = -6-4-6 = \underline{-16V}$

17

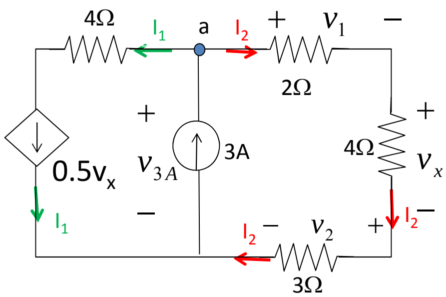
L4

Key point: Find a key variable so that you can express other variables in terms of it. Then you can use KVL, or KCL and ohms Law, to make an equation for it. For the previous circuit, I is the key variable.

Example: Find v_x and the power supplied by the 3A source.



Main idea:
Express branch currents in terms of v_x.
Then use KCL at node a to form an equation for v_x.



Q: How many different currents?

To find power supplied by 3A, assign v_{3A}. By KVL,

$v_{3A}=v_1+v_x+v_2 = 2I_2+v_x+3I_2$
$=2+4+3=9V$

Thus, $p_{3A}=-9\times3=-27W$.
Power supplied.

Question: Can you find v_{3A} from KVL along left side loop?

~~$4I_1 + 0.5v_x - v_{3A} = 0?$~~

$0.5v_x$ is a current. Huge mistake to put current in an KVL equation

Solution:
Assign current I_1 and I_2.

By KCL, same I_1 flows through upper 4Ω and dependent current source.
How to express I_1 in terms of v_x?

$I_1=0.5v_x$ (E1).

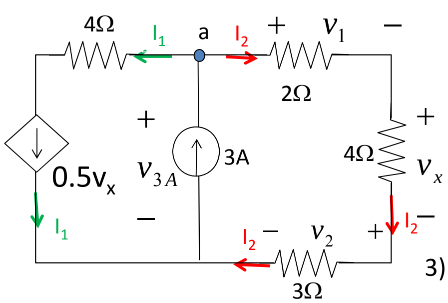
Also by KCL, same current I_2 flows through the 3 resistors in the right loop.
Thus by Ohm's law,

$I_2=v_x/4$ (E2)

Use KCL at node a, $I_1+I_2=3$.
Plug in (E1), (E2), to obtain
$0.5v_x+v_x/4=3, \Rightarrow v_x=4V$

With $v_x=4V$, have $I_2=v_x/4=1A$

Summary:



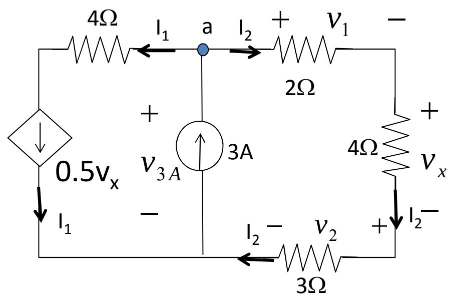
Main idea:
Express branch currents in terms of v_x.
Then use KCL at node a to form an equation for v_x.

Main steps:

1) Assign currents I_1 and I_2.
2) Use KCL at node a,
   $I_1+I_2=3$ (*)
3) Express I_1, I_2 in terms of v_x
   a) By property of the dependent source
      $I_1=0.5v_x$ (E1).
   b) By Ohm's law,
      $I_2=v_x/4$ (E2)
4) Plug (E1), (E2) in (*) to obtain
   $0.5v_x+v_x/4=3, \Rightarrow v_x=4V$

With v_x computed, you can find I_1, I_2,
And all resistor voltages and source voltages ,
And the power by each element

Another approach: use I_2 as key variable



KCL at a: $I_1 + I_2 = 3$ (E1)

$\Rightarrow$ $I_1 = 3 - I_2$

Another way to express I_1 in terms of I_2?

$I_1 = 0.5v_x \quad v_x = 4I_2,$

$\Rightarrow$ $I_1 = 0.5 \times 4I_2 = 2I_2$ (E2)

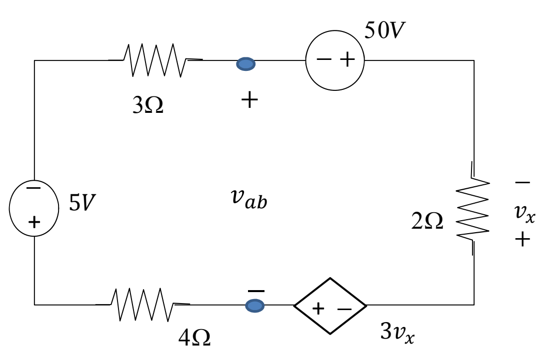
Plug (E2) into (E1)

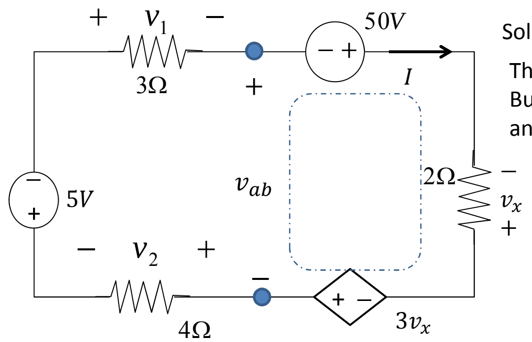
$2I_2 + I_2 = 3$

$I_2 = 1A$

$v_x = 4I_2 = 4V$

Example: Find $v_x,\ v_{ab}$





L4

Solution:

The loop current is not asked.
But you still need to use it as a key variable and make an equation for it using KVL.

Assign loop current I,

Assign resistor voltages $v_1,\ v_2$

Express v_1, v_2, v_x in terms of I:

$v_1 = 3I,\ v_2 = 4I\ \ ,\ v_x = -2I$ (E1)

Use KVL to make equation for the voltages:

$v_1 - 50 - v_x - 3v_x + v_2 + 5 = 0$ (E2)

To find v_{ab}, apply KVL on right-side loop

$$-50 - v_x - 3v_x - v_{ab} = 0$$

$$\begin{aligned} v_{ab} &= -50 - 4v_x \\ &= -50 - 4(-6) \\ &= -26V \end{aligned}$$

Plug (E1) into (E2) to obtain an equation for I:

$$3I - 50 - (-2I) - 3(-2I) + 4I + 5 = 0$$

$$3I - 50 + 2I + 6I + 4I + 5 = 0$$

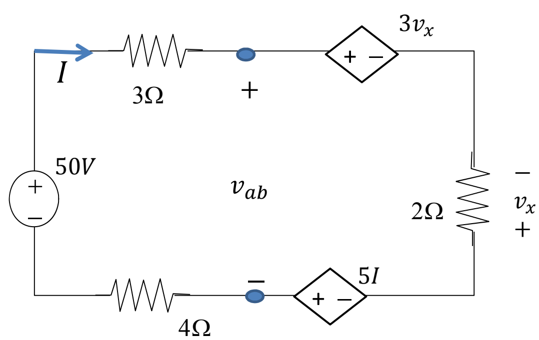
$15I = 45$,

$I = 3A$, by (E1)

$v_x = -2I = -2 \times 3 = -6V$

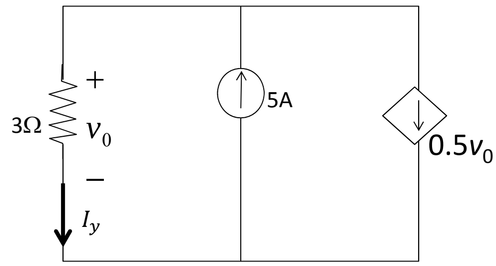
R4

Practice 3: Find $v_x,\ v_{ab}$



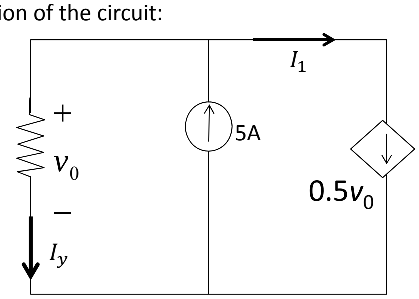
Practice 4: Find v_0 and I_y

R4

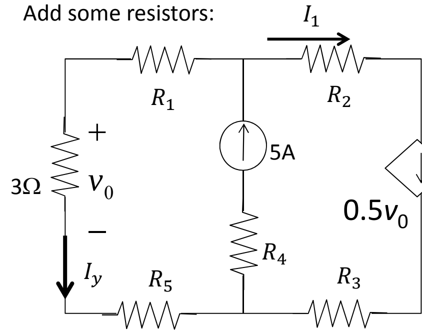


Variation of the circuit:

R4



Add some resistors:



For both circuits:

By KCL, $I_1 + I_y = 5$ (E1)

By the dependent current source,
$I_1 = 0.5v_0$ (E2)

By Ohm's Law:
$I_y = \frac{v_0}{3}$ (E3)

Plug (E2) and (E3) into (E1),
$0.5v_0 + \frac{v_0}{3} = 5$

➡ $v_0 = 6V$

$$I_y = \frac{v_0}{3} = \frac{6}{3} = 2A$$

You have the same relationship among $I_1, I_y, and\ v_0$.
Thus the answers are not changed.

But the power supplied by the sources will be different.

R4

Practice 5: Find v_0 and the power supplied by 6A source