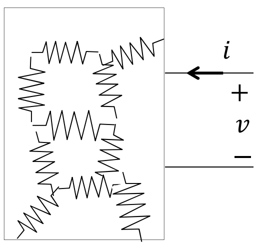
Today, we learn some tools derived from the 3 basic laws:

- Equivalent resistance
- Voltage division
- Current division

They help you develop intuition about circuits:

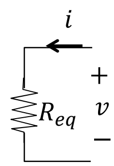
- To see relationship between variables
- To see several steps ahead
- To plan a solution to a circuit

Two terminal circuit
with all resistors



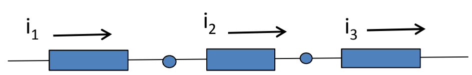
Behaves like a single resistor

$$R_{eq} \triangleq \frac{v}{i}$$



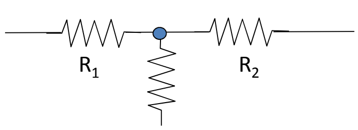
Ways of connection

Series connection: Two or more elements are in series if every connected pair exclusively share a single node, i.e., one node connects only two elements.



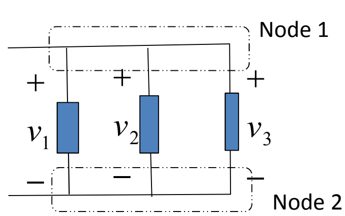
$i_1 = i_2 = i_3$

By KCL, same current flows through elements in series



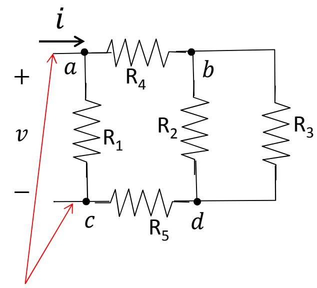
R_1 and R_2 are not in series because the node connecting them also connects a third element.

Parallel connection: Two or more elements are in parallel if they are connected between the same two nodes.



By KVL, same voltage across parallel elements:
$v_1 = v_2 = v_3$

Determine series and parallel connection:



R_2 and R_3 are in parallel,
How about R_1 and R_2?
R_1 and R_2 are not in parallel.
R_1 is between "a" and "c", R_2 between "b" and "d"
Any series connection?

R_1 and R_5 are not in series

R_1 and R_4 are not in series

No series connection between any two elements.

These wires cannot be thrown away.
They must be connected to somewhere to draw power

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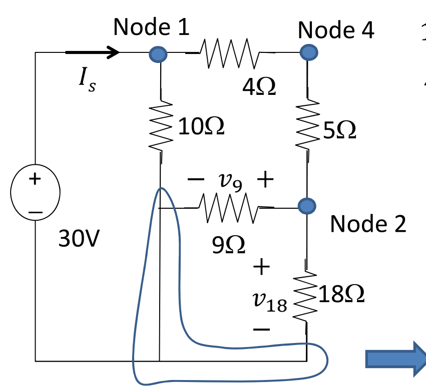
Which pair of resistors are in series? Which pair in parallel?



10Ω and 5Ω? Not parallel, Not series

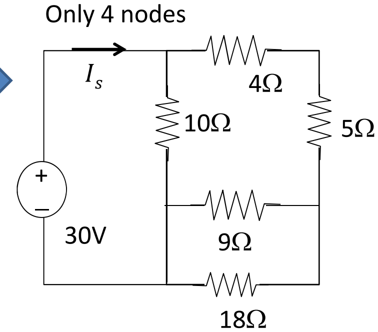
4Ω and 9Ω? Not parallel, Not series

4Ω and 5Ω? In Series

9Ω and 18Ω? In parallel

How many nodes in the circuit?

Only 4 nodes



$v_9 = v_{18}$

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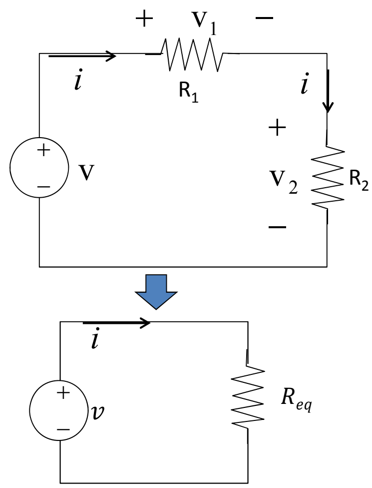
§ 2.5 Series resistors and voltage division



Given R_1, R_2.
What is v/i ?

By KCL, same current in R_1,R_2
By Ohm's law, $v_1 = R_1 i, v_2 = R_2 i$
By KVL, $v = v_1 + v_2$.
Putting together:
$v = v_1 + v_2 = R_1 i + R_2 i = (R_1 + R_2) i$
$\Rightarrow v = (R_1 + R_2) i$ (1)

$v = R_{eq} i$ (2) $R_{eq} =?$

Compare (1) and (2):

$$R_{eq} = R_1 + R_2$$

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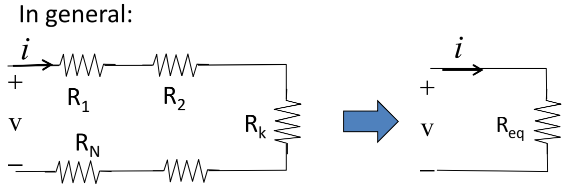
In general:



If $R_{eq}=R_1+R_2+....+R_N$, same v~ i relationship.
R_{eq} is called the equivalent resistance.

$R_{eq}=R_1+R_2+....+R_N$

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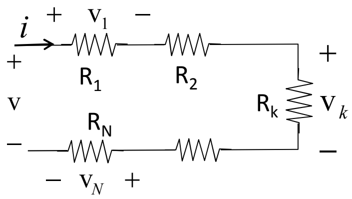
Voltage division

L5



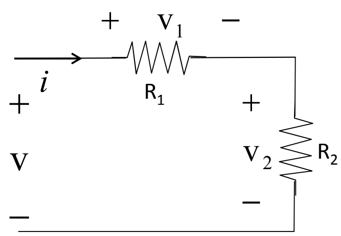
$$i = \frac{v}{R_1 + R_2 + \ldots + R_N}$$

$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + \ldots + R_N} v$$

$$v_k = R_k i = \frac{R_k}{R_1 + R_2 + \ldots + R_N} v$$

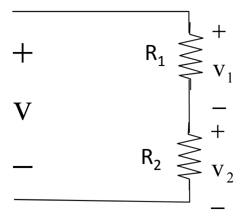
Voltage division: Total voltage v is divided among the resistors in direct proportion to the resistances. Larger resistance takes more voltage.

Special case with two resistors:



$$v_1 = \frac{R_1}{R_1 + R_2} v$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$



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§ 2.6 parallel resistors and current division

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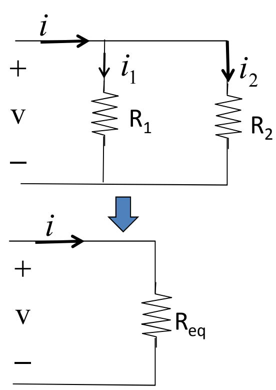

What is $R_{eq} = v/i$?

By KVL, same voltage across R_1, R_2
By ohm's law: $i_1 = v/R_1, i_2 = v/R_2$
By KCL: $i = i_1 + i_2$

Put together:

$$i = \frac{v}{R_1} + \frac{v}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v \qquad (1)$$

$$i = \frac{1}{R_{eq}} v \qquad (2)$$

$R_{eq} = ?$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

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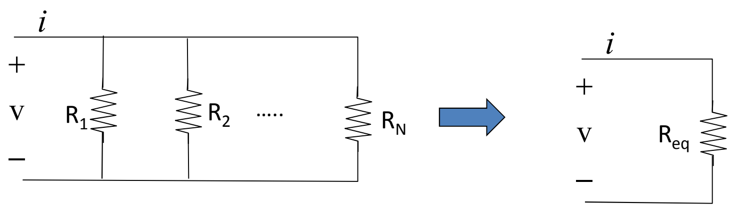
In general



$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}}$$

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Equivalent resistance for parallel resistors:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}}$$

Notation: $R_{eq} = R_1 // R_2 .... // R_N$

Special cases:

N=2: $R_1 // R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$

$R_1=R_2=\ldots=R_N=R$: $R_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{R} + \ldots + \frac{1}{R}} = \frac{R}{N}$

Simple combinations:

3//6= 2
12//6= 4
15//10= 6
20//5= 4

Simple rule:

$$\alpha R_1 // \alpha R_2 = \frac{\alpha R_1 \times \alpha R_2}{\alpha R_1 + \alpha R_2} = \frac{\alpha R_2 R_2}{R_1 + R_2} = \alpha (R_1 // R_2)$$

27//54= 9(3//6) $= 9 \times 2 = 18$

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Common sense:

Adding more resistor to existing parallel ones <u>reduces</u> R_{eq}:

$$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_N}+\frac{1}{R_{N+1}}} < \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_N}}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_N}} < \frac{1}{\frac{1}{R_k}} = R_k, \quad \text{for any } k = 1, 2, \ldots, N$$

Equivalent resistance for parallel connection is less than any individual resistance

Equivalent conductance:

$$G_{eq} = \frac{i}{v} = \frac{1}{R_{eq}} = \frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_N} = G_1 + G_2 + \ldots + G_N$$

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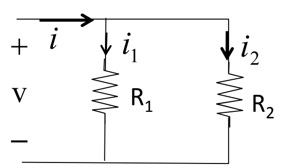
Current division: Given total i, what is i_1, i_2?



R_{eq}

$$i_1 = \frac{v}{R_1} = \frac{1}{R_1}\frac{R_1R_2}{R_1+R_2}i = \frac{R_2}{R_1+R_2}i$$

$$i_2 = \frac{v}{R_2} = \frac{1}{R_2}\frac{R_1R_2}{R_1+R_2}i = \frac{R_1}{R_1+R_2}i$$

$$v = \frac{R_1R_2}{R_1+R_2}i$$

$$i_1 = \frac{R_2}{R_1+R_2}i \qquad i_2 = \frac{R_1}{R_1+R_2}i$$

The other resistance on top

The current is shared by resistors in inverse proportion to resistance.
Larger resistor takes less current.

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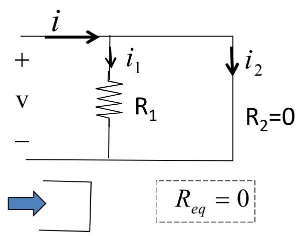
Two extreme cases:

Case 1: A resistor in parallel with a short circuit



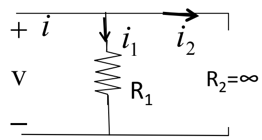
$$i_1 = \frac{R_2}{R_1 + R_2} i = \frac{0}{R_1 + 0} i = 0$$

$$i_2 = \frac{R_1}{R_1 + R_2} i = \frac{R_1}{R_1 + 0} i = i$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 \times 0}{R_1 + 0} = 0$$

$$R_{eq} = 0$$

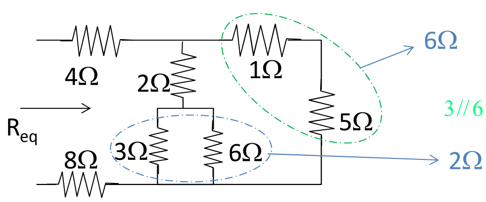
Case 2: A resistor in parallel with an open circuit



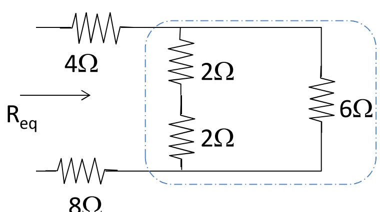
$$i_1 = i$$
$$i_2 = 0$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{\infty}} = R_1$$

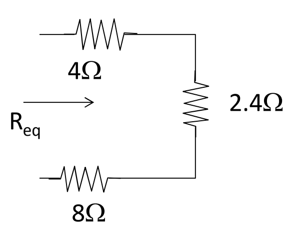
Equivalent resistance Examples



$$3//6 = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega$$



$$(2+2)//6 = \frac{4 \times 6}{4+6} = \frac{24}{10} = 2.4\Omega$$



R_{eq}=4+2.4+8=14.4Ω

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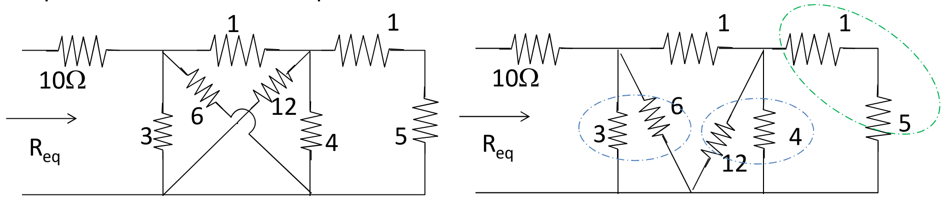
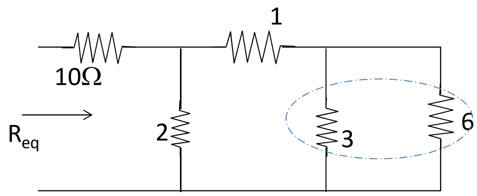
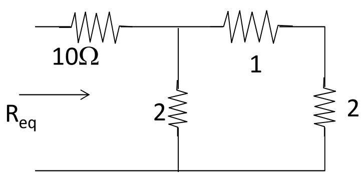
Equivalent resistance Examples







R_{eq}=10+2//(1+2)
=10+2//3
=10+1.2
=11.2Ω

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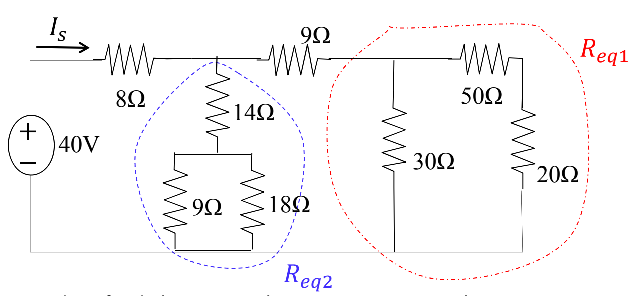
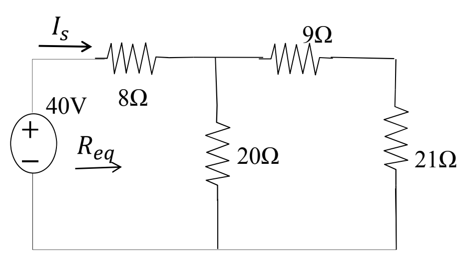
Example: Compute I_s



R_{eq1} = 30//(20+50)
= 30//70
= 21Ω

R_{eq2} = 14+9//18
= 14 + 6
= 20Ω

Need to find the equivalent resistance with respect to 40V



R_{eq} = 8 + 20//(21+9)
= 8+12 =20Ω

$$I_s = \frac{40}{20} = 2A$$

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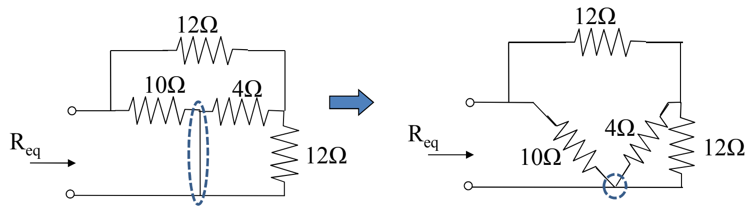
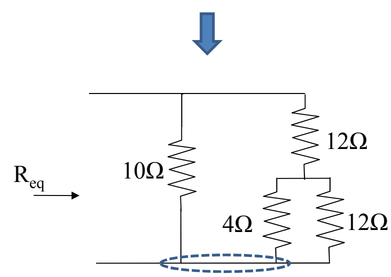
Example: Compute R_{eq}



R_{eq} = 10// (12+4//12)
= 10 // 15 =6 Ω



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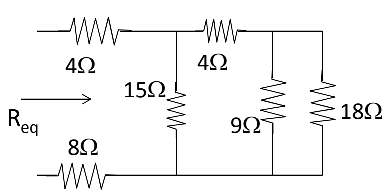
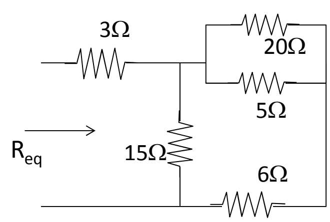
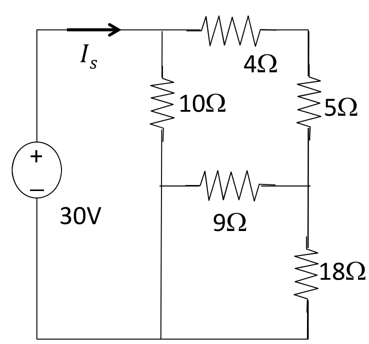
R5

Practice 6: Find R_{eq}





Practice 7: Find I_s



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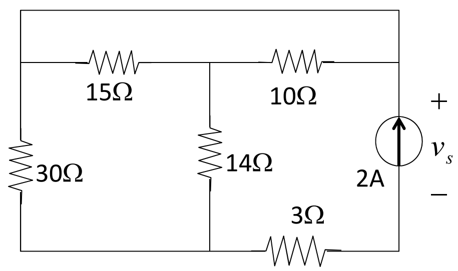
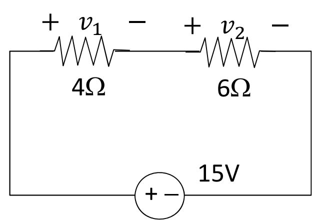
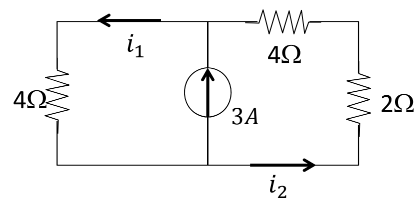
Practice 8: Find v_s



Practice 9: Find v_1, v_2



Practice 10: Find i_1, i_2



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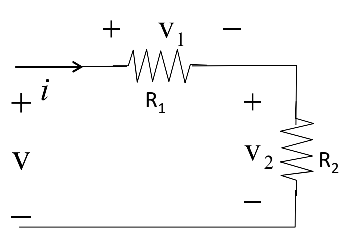
Three useful tools derived from basic laws:

- Equivalent resistance
- Voltage division
- Current division

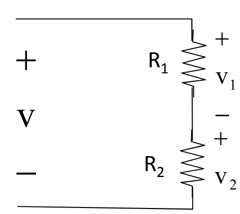
Used together to solve circuit problems

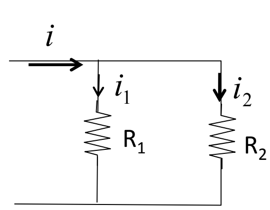


Voltage division:

$$v_1 = \frac{R_1}{R_1 + R_2} v$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

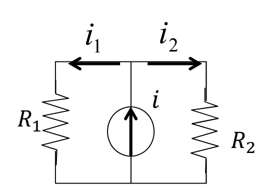




Current Division:

$$i_1 = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = \frac{R_1}{R_1 + R_2} i$$



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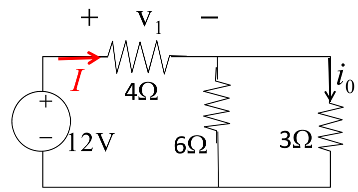
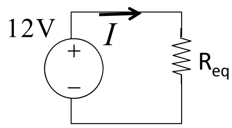
Example: Find i_0 and v_1.



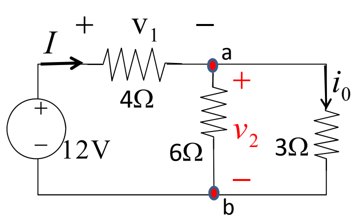
Approach 1: Assign auxiliary variable I.

Use equivalent resistance with respect to 12V to find I, then v_1=4I, and i_0 can be computed by current division.

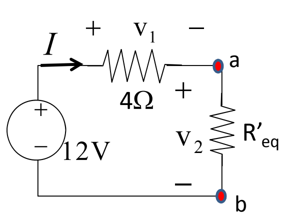


R_{eq}= 4 + 6//3 =4+2=6Ω
By Ohm's Law, I=12/R_{eq}=12/6=2A
Thus v_1= 4I=8V.
By current division: $i_0 = \frac{6}{3+6} I = \frac{6}{3+6} \times 2 = \frac{4}{3} A$

Be Careful: In this equivalent circuit, only I is the same as in the original circuit. Neither v_1 nor i_0, can be found in it. You need to go back to the original circuit to find v_1 and i_0.



Approach 2: Use equivalent resistance of 6//3, denoted as R'_{eq}.



R'_{eq}=3//6=2Ω. By voltage division,

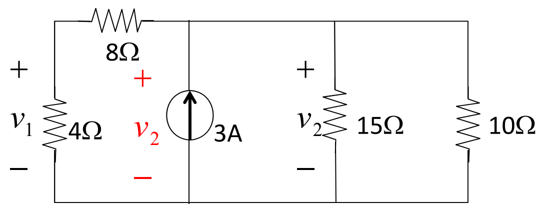
$$v_1 = \frac{4}{4+2} \times 12 = 8V, \; v_2 = \frac{2}{4+2} \times 12 = 4V$$

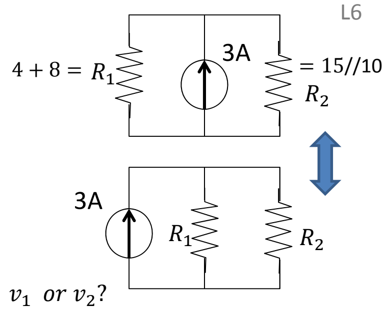
Be careful, i_0 cannot be found in the equivalent circuit. $i_0 \neq I$. You have to use the original circuit to find i_0.

Where is v_2 in the original circuit?

Since the voltage across 3Ω is v_2, by Ohm's Law, $i_0 = v_2/3 = 4/3$A

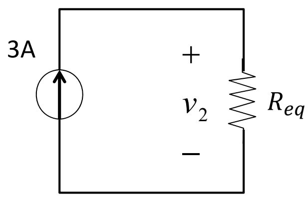
Example: Find v_1, v_2,





Approach 1: Which is the voltage across 3A ? v_1 *or* v_2?

It is v_2. If you know the Req w.r.t 3A, you can obtain v_2 by ohm's law.



$R_{eq} = R_1 // R_2$

R_{eq}= (4+8)//15//10

= 12//6

= 4Ω

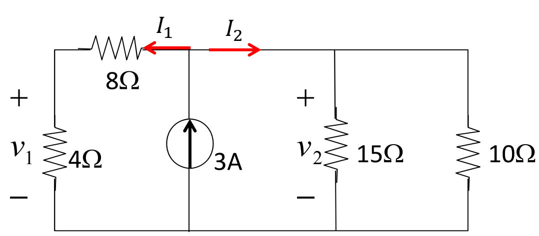
$v_2 = R_{eq} \times 3$
$= 4 \times 3 = 12\text{V}$

How to find v_1?

Need the original circuit.

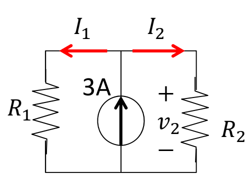
By voltage division,

$$v_1 = \frac{4}{4+8} \times v_2 = \frac{4}{12} \times 12 = 4V$$



Be careful, you cannot find v_1 in the simplified circuit. Have to use the original circuit to find v_1

Approach 2: Use current division. Assign I_1, I_2.



$R_1 = 4 + 8 = 12\Omega$

R_2 =15//10=6Ω

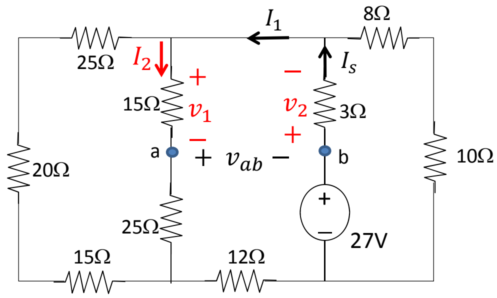
By current division,

$$I_1 = \frac{R_2}{R_1 + R_2} \times 3 = \frac{6}{18} \times 3 = 1A$$

$$I_2 = \frac{R_1}{R_1 + R_2} \times 3 = \frac{12}{18} \times 3 = 2A$$

By ohm's law, $v_1 = 4I_1 = 4V$; $v_2 = R_2 I_2 = 6 \times 2 = 12V$.

Example: Find $I_s,\ I_1,\ v_{ab}$



Outline of the solution:

Find I_s first, then use current division to find I_1.

To find v_{ab}, need to find v_1, v_2, then use KVL

$$v_2 + v_{ab} + v_1 = 0 \Rightarrow v_{ab} = -v_2 - v_1$$

To find v_1, need the current through 15Ω.

Assign I_2,

I_2 can be computed by current division on I_1

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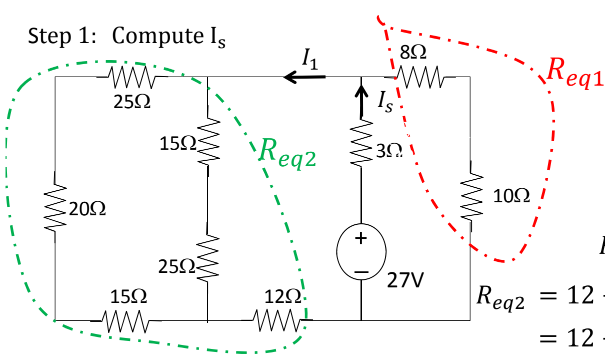
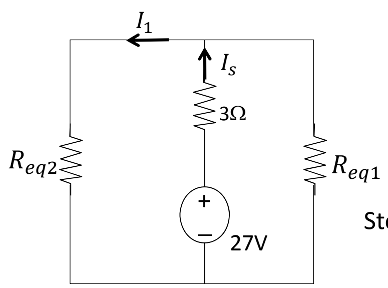
Step 1: Compute I_s





$$R_{eq1} = 10 + 8 = 18\Omega$$

$$R_{eq2} = 12 + (15 + 25)\backslash\backslash(25 + 20 + 15)$$
$$= 12 + 40\backslash\backslash\ 60 = 12 + 24 = 36\Omega$$

R_{eq} w.r.t. 27V:

$$R_{eq} = 3 + R_{eq1}\backslash\backslash\ R_{eq2} = 3 + 36\backslash\backslash 18$$
$$= 3 + 6(6\backslash\backslash 3) = 3 + 6 \times 2 = 15\Omega$$

$$I_s = \frac{27}{R_{eq}} = \frac{27}{15} = 1.8A$$

Step 2: By current division: $I_1 = \dfrac{R_{eq1}}{R_{eq1} + R_{eq2}} \times I_s$

$$= \frac{18}{18 + 36} \times 1.8 = 0.6A$$

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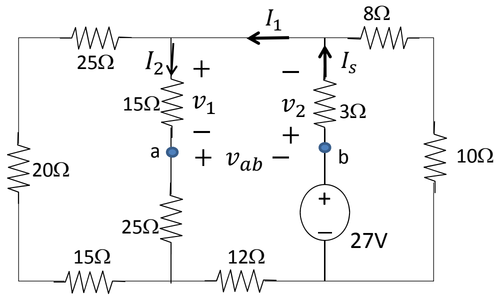
Step 3: Go back to the original circuit to find v_{ab}:



$I_s = 1.8A;$
$I_1 = 0.6A$

I_1 is divided by $15\Omega + 25\Omega = 40\Omega$ and $25\Omega + 20\Omega + 15\Omega = 60\Omega$

$$I_2 = \frac{60}{60+40} \times 0.6 = 0.36A$$

$v_1 = 15I_2 = 15 \times 0.36 = 5.4V$

$v_2 = 3I_s = 3 \times 1.8 = 5.4V$

$v_{ab} + v_1 + v_2 = 0$

$v_{ab} = -v_1 - v_2 = -5.4 - 5.4 = -10.8V$

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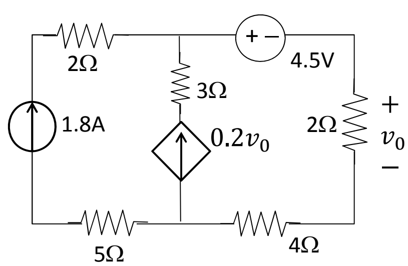
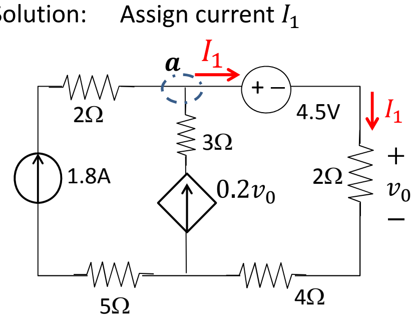
Example: Find v_0 and the power absorbed by the dependent current source.



Hint: *Use KCL to make an equation for* v_0

Solution: Assign current I_1



I_1 is the same current through 2Ω

By Ohm's Law, $I_1 = \frac{v_0}{2} = 0.5v_0$

KCL at node $\boldsymbol{a}$:

$1.8 + 0.2v_0 = I_1 = 0.5v_0$

$\Rightarrow 1.8 = 0.3v_0$

$\Rightarrow v_0 = 6V$

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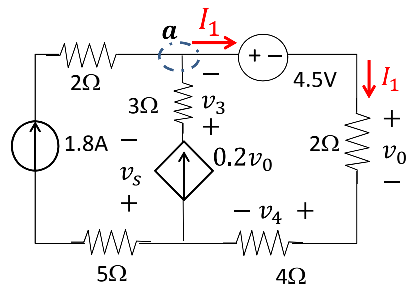
We have : $v_0 = 6V$



Power absorbed by the **dependent current source**?

Need voltage across $0.2v_0$

Assign v_s

To apply KVL correctly,
Assign v_3, v_4

$v_3 = ?$ $\quad v_3 = 3 \times 0.2v_0$

$= 3 \times 0.2 \times 6 = 3.6V$

$v_4 = ?$ $\quad v_4 = 4I_1, \quad I_1 = \dfrac{v_0}{2} = \dfrac{6}{2} = 3A$

$v_4 = 4 \times 3 = 12V$

KVL around right side loop:

$v_s + v_3 + 4.5 + v_0 + v_4 = 0$

$v_s = -v_3 - 4.5 - v_0 - v_4$

$= -3.6 - 4.5 - 6 - 12$

$= -26.1V$

$p_{0.2v_0} = \mathrm{v_s} \times 0.2v_0$

$= -26.1 \times 0.2 \times 6$

$= -31.32W$

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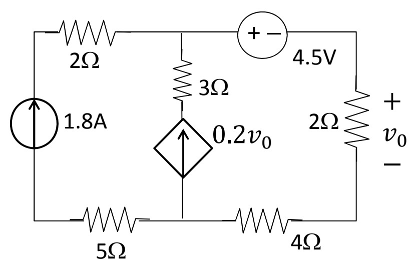
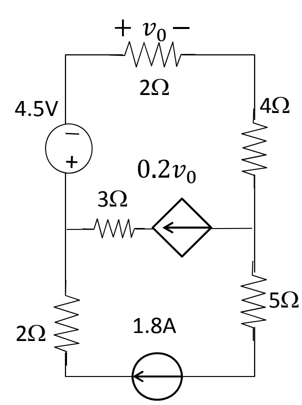
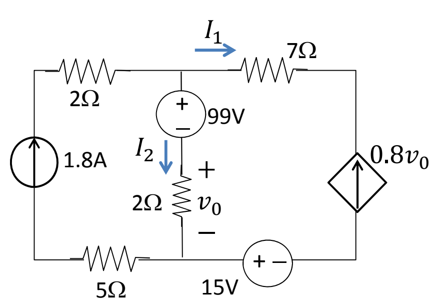
Variations of the circuit







Same idea!
Express Branch current I_1, I_2
In terms of v_0,
Then apply KCL

If power by a current source is needed, compute its voltage by using KVL

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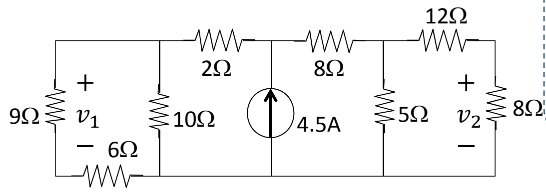
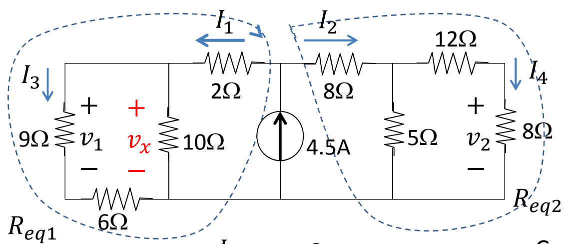
Example: Find v_1 and v_2.

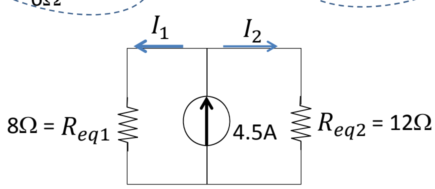


Alternatively:
Ohms Law: $v_x = 6I_1 = 16.2V$
Voltage division:
$v_1 = \frac{9}{9+6} \times v_x$ =9.72V



More current division:

$$I_3 = \frac{10}{10+15} \times I_1 = 1.08A$$
$$v_1 = 9I_3 = 9.72V$$
$$I_4 = \frac{5}{5+20} \times I_2 = 0.36A$$
$$v_2 = 8I_4 = 2.88V$$



Current division:

$$I_1 = \frac{12}{8+12} \times 4.5 = 2.7A$$
$$I_2 = \frac{8}{8+12} \times 4.5 = 1.8A$$

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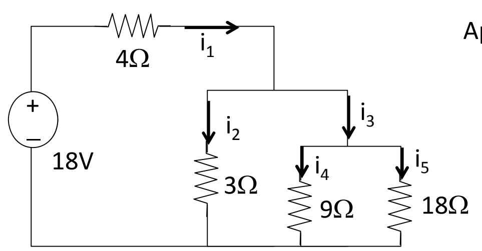
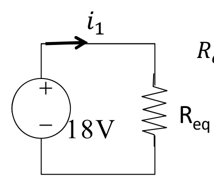
Example: Find the currents i_1, …, i_5

L6

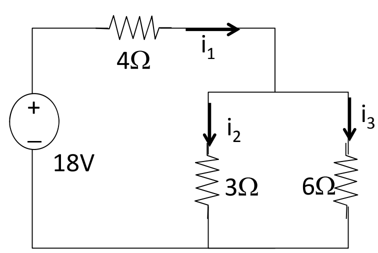


Approach1: Use equivalence resistance and current division.



$R_{eq} = 4$ +3//9//18=6Ω

$$i_1 = \frac{18}{6} = 3A$$



$$i_2 = \frac{6}{3+6} \times i_1 = \frac{6}{9} \times 3 = 2A$$
$$i_3 = \frac{3}{3+6} \times i_1 = \frac{3}{9} \times 3 = 1A$$
$$i_4 = \frac{18}{9+18} \times i_3 = \frac{18}{27} \times 1 = \frac{2}{3}A$$
$$i_5 = \frac{9}{9+18} \times i_3 = \frac{9}{27} \times 1 = \frac{1}{3}A$$

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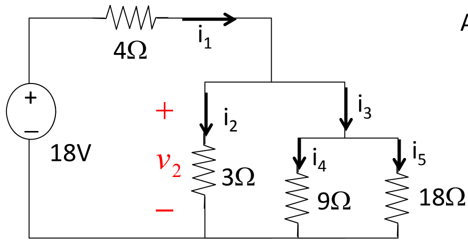
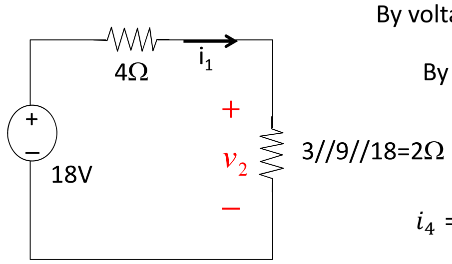
L6

Example: Find the currents $i_1, \ldots, i_5$



Approach2: Use voltage division and Ohm's law. Assign v_2.



By voltage division $v_2 = \frac{2}{4+2} \times 18 = 6V$

By Ohm's law, $i_1 = \frac{v_2}{2} = \frac{6}{2} = 3A$

$$i_2 = \frac{v_2}{3} = \frac{6}{3} = 2A$$

$$i_4 = \frac{v_2}{9} = \frac{6}{9} = \frac{2}{3}A \qquad i_5 = \frac{v_2}{18} = \frac{6}{18} = \frac{1}{3}A$$

$$i_3 = i_4 + i_5 = \frac{2}{3} + \frac{1}{3} = 1A$$

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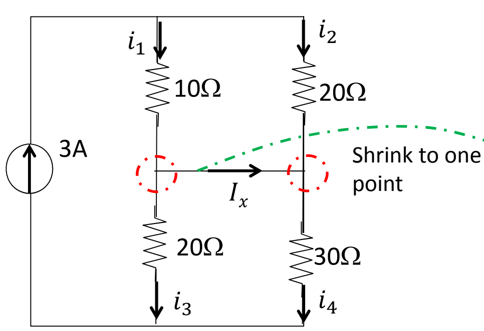
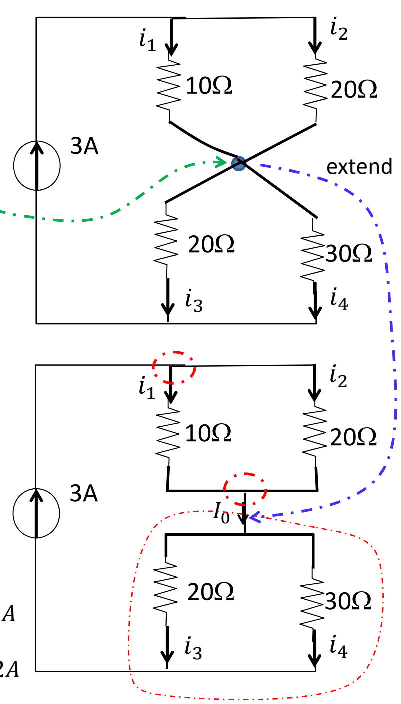
Example: Find i_1, i_2, i_3, i_4, I_x





In all three circuits, i_1, i_2, i_3, i_4 are the same

How about I_x, I_0? By KCL, $I_0 = i_1 + i_2$=?

$$I_0 = i_1 + i_2 = i_3 + i_4 = 3A$$

$$I_x = i_1 - i_3 = i_4 - i_2$$

By current division:

$i_1 = \frac{20}{30} \times 3 = 2A; i_2 = 1A;$ $\quad I_x = i_1 - i_3 = 0.2A$

$i_3 = 1.8A; i_4 = 1.2A;$ $\quad I_x = i_4 - i_2 = 0.2A$

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- Test 1 will be given on Sept 30 (Monday), 11-11:50am.
  In Ball Hall 210 There will be two versions in different colors.

  Please arrive 5-10 minutes earlier

- A practice exam will be given on 9/25/2019(Wednesday), 11-11:50am
  in Ball Hall 210

Solution to practice exams will be posted at website
Solution to all practice problems in lecture note will be posted.

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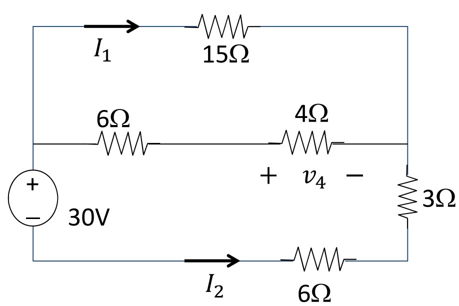
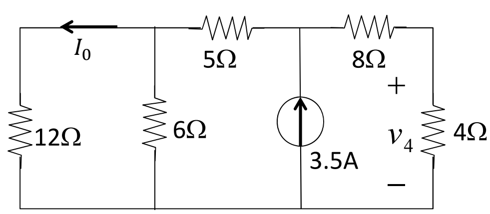
R6

Practice 12: Find $I_1,\ I_2,\ v_4$



Practice 13: Find I_0, v_4



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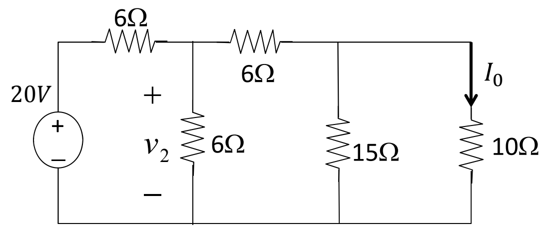
R6

Practice 14: Find I_0, v_2



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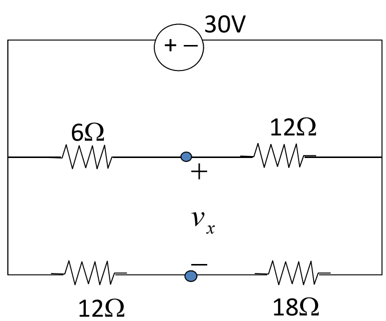
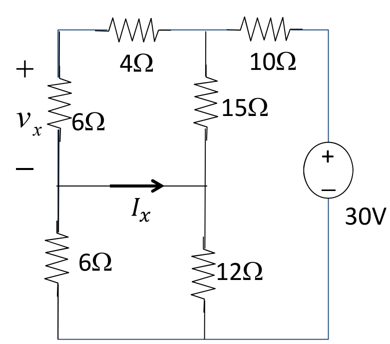
R6

Practice 15: Find the voltage v_x



Practice 16: Find v_0, I_x,



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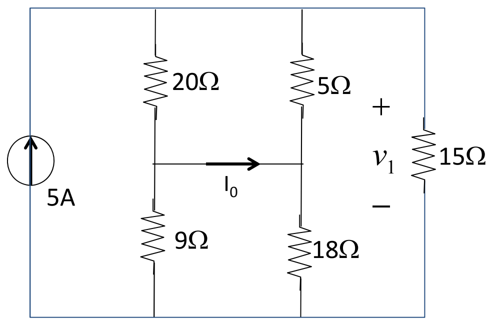
R6

Practice 16a: Find v_1, I_0,



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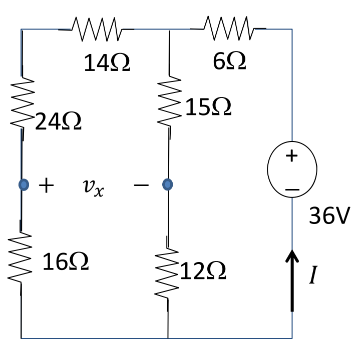
R7

Practice 17: Find $v_x, I,$



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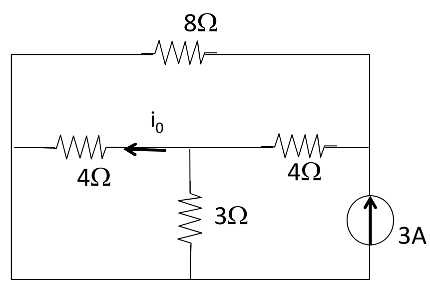
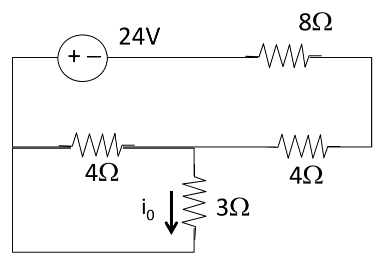
R7

Practice 18: Find i_0



Practice 19: Find i_0



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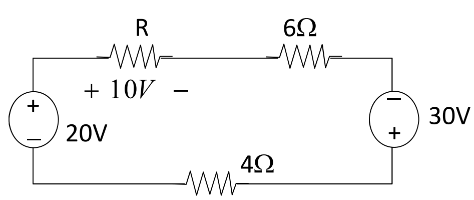
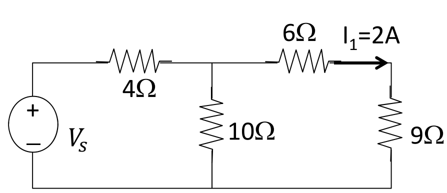
R7

Practice 20: Find R for the circuit



Practice 21: Find V_s for the circuit



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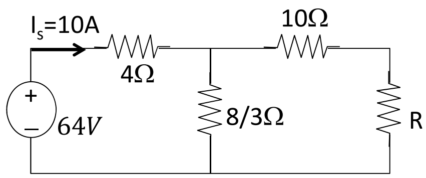
R7

Practice 22: Find R so that I_s is 10A



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