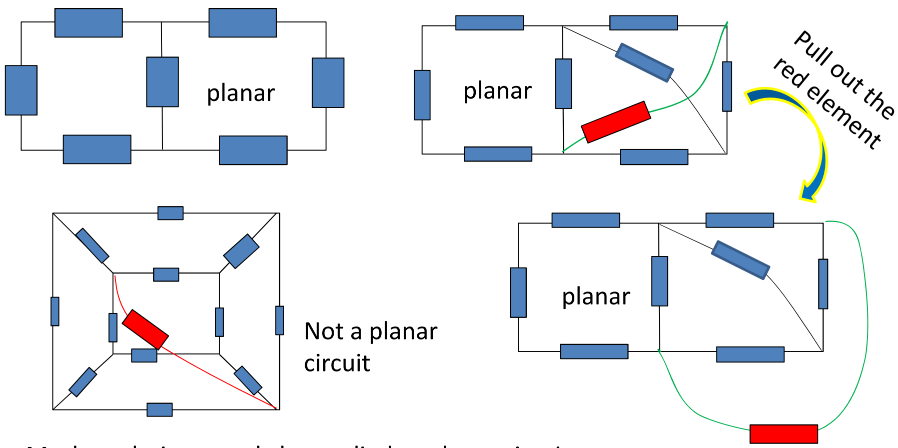
§ 3.4 Mesh analysis - for circuits without current sources

Planar circuit: A circuit that can be drawn in a plane with no branches crossing each other



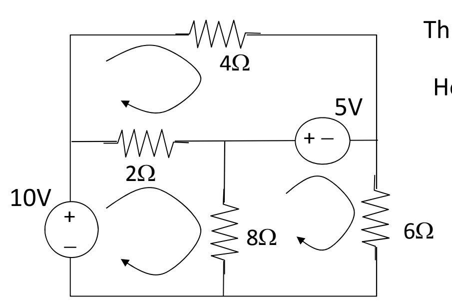

Mesh analysis can only be applied to planar circuits

Nodal analysis can be applied to any circuit.



Basic concepts: Mesh and mesh current

Mesh: a loop that does not contain any other loop in it

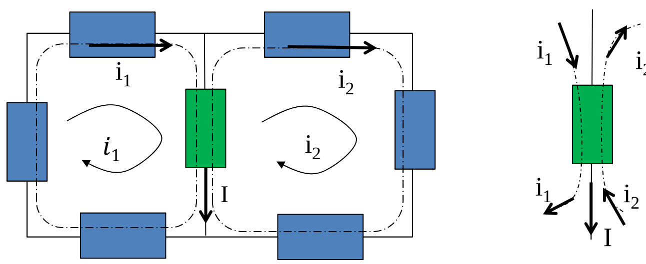



This circuit has 3 meshes

How many loops?

Mesh Current: Mesh currents are variables assigned to all meshes in a certain direction, so that the current through each element is the algebraic sum of two mesh currents.

For convenience, choose clockwise direction



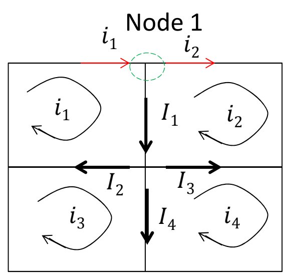

$I = i_1 - i_2$

Same direction as I

Opposite direction to I



In a planar circuit, an element is involved in at most two meshes




$I_1 = i_1 - i_2$; ← KCL at node 1

$I_2 = i_1 - i_3$

$I_3 = i_4 - i_2$;

$I_4 = i_3 - i_4$

Sum of current entering the center node:

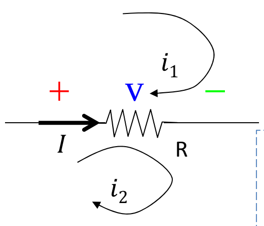
$$I_1 - I_2 - I_3 - I_4$$
$$= i_1 - i_2 - i_1 + i_3 - i_4 + i_2 - i_3 + i_4$$
$$= 0$$

KCL automatically satisfied

Basic Idea of mesh analysis:

Express branch voltages in terms of mesh currents.
Form equations by applying KVL along the meshes.

Basic step:
Express resistor voltages in terms of mesh currents




$I = i_2 - i_1 \qquad v = RI = R\,(i_2 - i_1)$

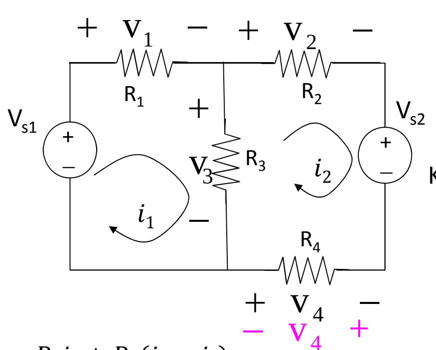
Later, " $I$ " will not be assigned.

$v = R\,(i_2 - i_1)$

Enters at + (for $i_2$) Enters at − (for $i_1$)



Example: Find the mesh currents $i_1, i_2$




$v_1 = R_1 i_1 \qquad v_2 = R_2 i_2$

$v_3 = R_3(i_1 - i_2)$

$v_4 = -R_4 i_2$

$v_4 = R_4 i_2$

KVL along mesh 1:

$v_1 + v_3 - V_{s1} = 0$ (1)

KVL along mesh 2:

$v_2 + V_{s2} - v_4 - v_3 = 0$ (2)

$v_2 + V_{s2} + v_4 - v_3 = 0$ (2)

$R_1 i_1 + R_3(i_1 - i_2) = V_{s1}$

$R_2 i_2 - (-R_4 i_2) - R_3(i_1 - i_2) = -V_{s2}$

$R_1 i_1 + R_3(i_1 - i_2) = V_{s1}$

$R_2 i_2 + R_4 i_2 - R_3\,(i_1 - i_2) = -V_{s2}$

$(R_1 + R_3) i_1 - R_3 i_2 = V_{s1}$

$-R_3 i_1 + (R_2 + R_3 + R_4)\, i_2 = -V_{s2}$

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_{s1} \\ -V_{s2} \end{bmatrix}$$

Summary of general steps for circuit without current sources

Step 1: Assign mesh currents, $i_1, i_2, \ldots, i_n$

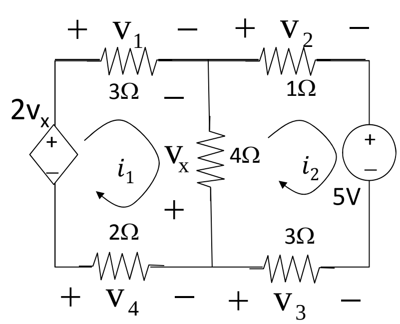
Step 2: Assign resistor voltages with polarity, and express these voltages in terms of mesh currents

Step 3: Apply KVL along each mesh to obtain n equations with respect to mesh currents (first in terms of voltages, then plug in the expressions from step 2)

Step 4: Solve equations for mesh currents. Obtain everything else asked



Example: Find $i_1, i_2$




Express resistor voltages

$v_1 = 3i_1;\ \ v_2 = i_2;$

$v_3 = -3i_2;\ \ v_4 = -2i_1$ ← Why "-"?

$v_x = 4(i_2 - i_1)$

KVL along mesh 1:

$v_1 - v_x - v_4 - 2v_x = 0$

$3i_1 - 4(i_2 - i_1) - (-2i_1) - 2\times 4(i_2 - i_1) = 0$

$17i_1 - 12i_2 = 0 \quad (1)$

KVL along mesh 2:

$v_2 + 5 - v_3 + v_x = 0$

$i_2 + 5 - (-3i_2) + 4(i_2 - i_1) = 0$

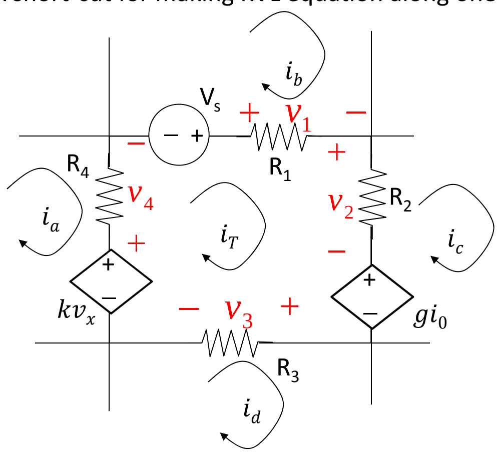
$-4i_1 + 8i_2 = -5 \quad (2)$

Solving (1) and (2): $i_1 = -0.682A$

$i_2 = -0.966A$

A short-cut for making KVL equation along one mesh:

L10




Consider the center mesh
We call it "This mesh"
All resistor voltages are assigned as voltage drop along clockwise direction.

$$v_1 = R_1(i_T - i_b)$$
$$v_2 = R_2(i_T - i_c)$$
$$v_3 = R_3(i_T - i_d)$$
$$v_4 = R_2(i_T - i_a)$$

The current of "This mesh", $i_T$ has positive sign,
Other mesh currents all have negative sign.

KVL along "This mesh": $-V_s + v_1 + v_2 + gi_0 + v_3 - kv_x + v_4 = 0$

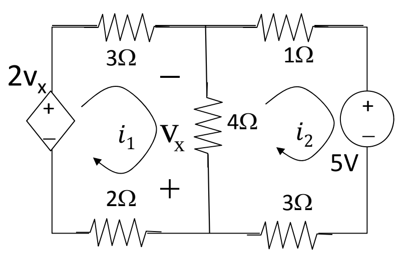
All resistor voltages have positive signs. Plug in the expression:

$$-V_s + R_1(i_T - i_b) + R_2(i_T - i_c) + gi_0 + R_3(i_T - i_d) - kv_x + R_4(i_T - i_a) = 0$$

In the future, you don't need to assign resistor voltages.
You can directly write the equation. The simple rule is that
voltage drop of $R_k = R_k$ (current of This mesh – current of other mesh)

L10

Previous example:




You must express controlling voltage or current: $v_x = 4(i_2 - i_1)$

Clockwise along mesh 1:

Voltage drop across 3Ω: $3i_1$
Voltage drop across 4Ω: $4(i_1 - i_2)$
Voltage drop across 2Ω: $2i_1$

KVL along mesh 1:
$$3i_1 + 4(i_1 - i_2) + 2i_1 - 2v_x = 0$$

Clockwise along mesh 2:

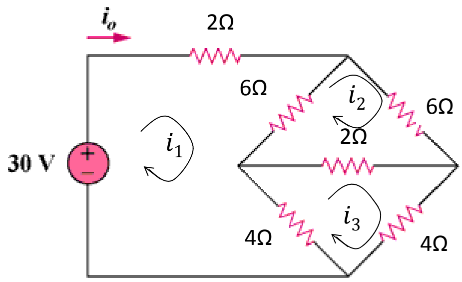
Voltage drop across 1Ω: $i_2$
Voltage drop across 3Ω: $3i_2$
Voltage drop across 4Ω: $4(i_2 - i_1)$

KVL along mesh 2:
$$i_2 + 5 + 3i_2 + 4(i_2 - i_1) = 0$$

Plug in $v_x$ and simplify:
$$17i_1 - 12i_2 = 0$$
$$-4i_1 + 8i_2 = -5$$

R10

Practice 1: Form 3 mesh equations for $i_1, i_2, i_3$

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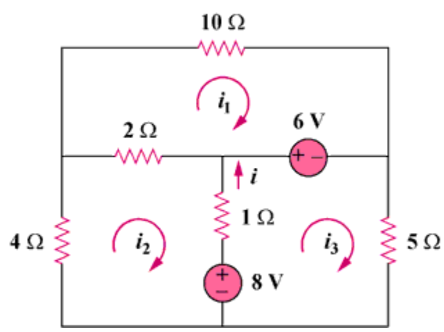
Practice 2: Form 3 mesh equations for $i_1, i_2, i_3$



Practice 3: Form 3 mesh equations for $i_1, i_2, i_3$

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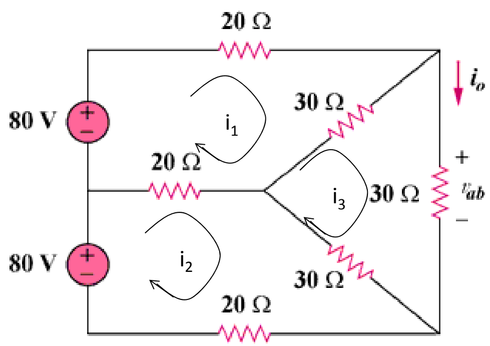
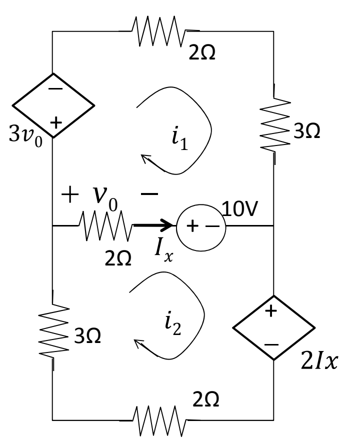



R10

Practice 4: Find $i_1, i_2$ and $v_0$



## § 3.5 Mesh analysis with current source

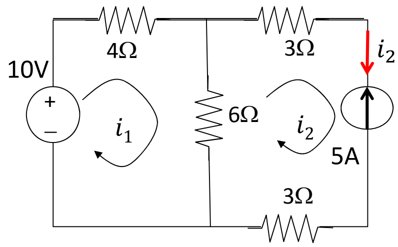


What is the trouble with current source?

Its voltage cannot be expressed in terms of mesh currents

Posing problems for making equations by KVL

Case 1: The current source exists only in one mesh
-- at the boundary of the circuit




What is $i_2$? $i_2 = -5A$

We have only one unknown mesh current $i_1$

One equation is enough

KVL along mesh 1 $4i_1 + 6(i_1 - i_2) = 10$ ← $i_2 = -5A$

$4i_1 + 6(i_1 + 5) = 10, \quad 10i_1 = -20; \quad i_1 = -2A$

Case 2: The current source exists as a common branch of two meshes

L11

All resistor voltages

$v_1 = 6i_1; \; v_2 = 2(i_1 - i_2)$
$v_3 = 10i_2; \; v_4 = 4i_2$

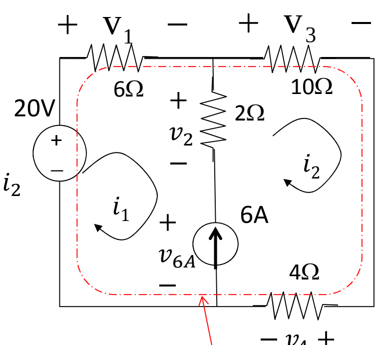
How to express $v_{6A}$? Cannot be expressed directly in terms of $i_1, i_2$

KVL along mesh 1:

$$v_1 + v_2 + v_{6A} = 20 \quad (1)$$

KVL along mesh 2:

$$v_3 + v_4 - v_{6A} - v_2 = 0 \quad (2)$$




Both equations has $v_{6A}$ (and $v_2$) but with different signs.

Adding up (1) and (2): $v_1 + v_3 + v_4 = 20$ (3)

What is (3)? It is the KVL along the outer loop. Called a super mesh

In the future, we don't make KVL along individual meshes (with current sources) and add them up.

We make KVL along a super mesh directly.

Super mesh: is formed by combining two or more meshes

All resistor voltages

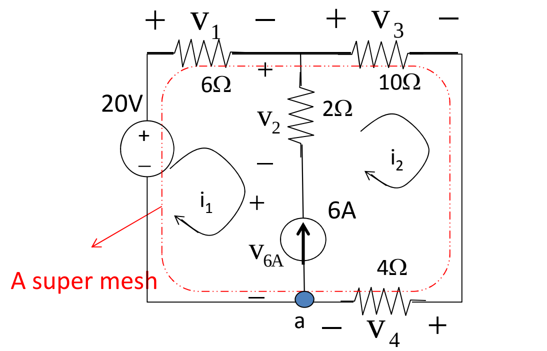
$v_1 = 6i_1;\ \ v_2 = 2(i_1 - i_2)$
$v_3 = 10i_2;\ v_4 = 4i_2$

KVL along super mesh:

$$v_1 + v_3 + v_4 = 20 \qquad (3)$$

Plug in voltage expressions:

$$6i_1 + 10i_2 + 4i_2 = 20 \qquad \text{(Eq1)}$$




Need another equation for $i_1$ and $i_2$.

It comes from the 6A current source:

$$i_2 - i_1 = 6 \qquad \text{(Eq2)}$$

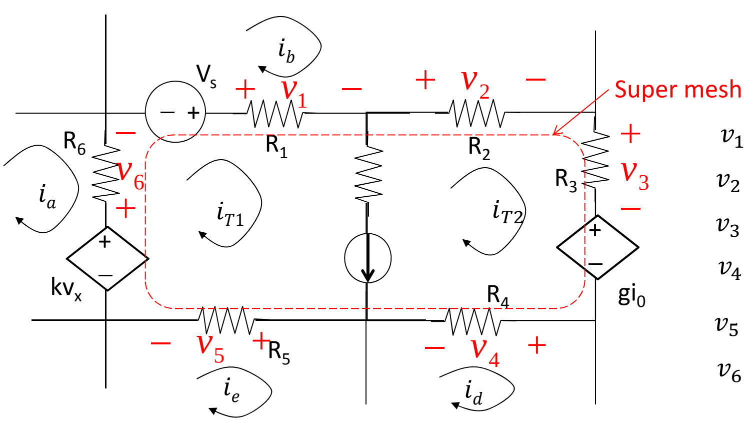
You can also consider (Eq2) as KCL at node a.

Solving (Eq1) and (Eq2) to obtain:
$i_1 = -3.2A;\ \ i_2 = 2.8A$

The purpose of making a super mesh is to avoid the voltage of a current source in a KVL equation.



A short-cut for making KVL equation along super mesh:




$v_1 = R_1(i_{T1} - i_b)$
$v_2 = R_2(i_{T2} - i_b)$
$v_3 = R_3 i_{T2}$
$v_4 = R_4(i_{T2} - i_d)$
$v_5 = R_5(i_{T1} - i_e)$
$v_6 = R_6(i_{T1} - i_a)$

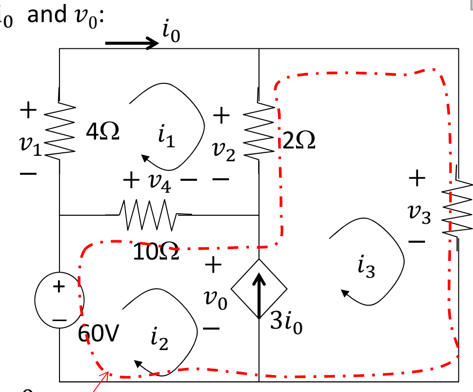
Let $i_{T1}$ $i_{T2}, i_{T3}$, … be the mesh currents inside the super mesh. Voltage drop of a resistor along clockwise direction: $v = R(i_{Tk} -$ other mesh current),
$i_{Tk}$ is one of $i_{T1}$, $i_{T2}$, … For example, across $R_1$: $v_1 = R_1(i_{T1} - i_b)$

KVL along super mesh: $-V_s + V_1 + V_2 + V_3 + gi_0 + V_4 + V_5 - kv_x + V_6 = 0$

$$-V_s + R_1(i_{T1} - i_b) + R_2(i_{T2} - i_b) + R_3 i_{T2} + gi_0 + R_4(i_{T2} - i_d) + R_5(i_{T1} - i_e) - kv_x + R_6(i_{T1} - i_a) = 0 \qquad (1)$$

L11

Example: Use mesh analysis to find $i_0$ and $v_0$:




Solution 1:

Assign mesh currents, i1, i2, i3

Assign resistor voltages: v1, v2, v3, v4

Express resistor voltages:

$v_1 = -4i_1 \qquad v_2 = 2(i_1 - i_3)$

$v_3 = 8i_3 \qquad v_4 = 10(i_2 - i_1)$

KVL along mesh 1:

$-v_1 + v_2 - v_4 = 0$

$-(-4i_1) + 2(i_1 - i_3) - 10(i_2 - i_1) = 0$

$16i_1 - 10i_2 - 2i_3 = 0 \qquad (1)$

Due to the current source $3i_0$, need to combine mesh 2 and mesh 3 into a super mesh

KVL along super mesh:

$v_4 - v_2 + v_3 = 60$

$10(i_2 - i_1) - 2(i_1 - i_3) + 8i_3 = 60$

$-12i_1 + 10i_2 + 10i_3 = 60 \qquad (2)$

By the $3i_0$ current source:

$i_3 - i_2 = 3i_0 \qquad \Leftarrow \quad i_0 = i_1$

$i_3 - i_2 = 3i_1$

$3i_1 + i_2 - i_3 = 0 \qquad (3)$

Solving (1),(2),(3) $\quad i_0 = i_1 = 1.731A,$

$i_1 = 1.731A, i_2 = 1.442A; i_3 = 6.635A$

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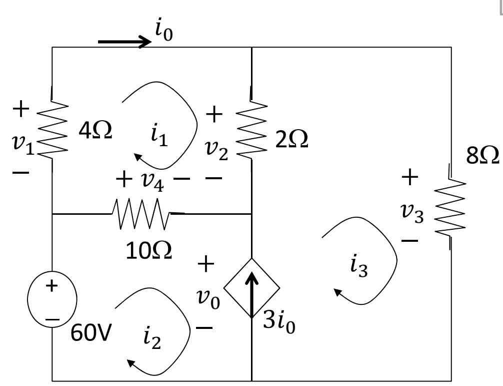



We have found

$i_1 = 1.731A,$
$i_2 = 1.442A$
$i_3 = 6.635A$

You can use these currents to find all resistor voltages.

Question: How to compute $v_0$?

Answer: By KVL

KVL along mesh 2,

$v_4 + v_0 - 60 = 0$

$v_0 = 60 - v_4 \qquad v_4 = 10(i_2 - i_1) = -2.89V$

$v_0 = 62.89V$

Solution 2:
Using shortcut to make KVL equations:

KVL along mesh 1

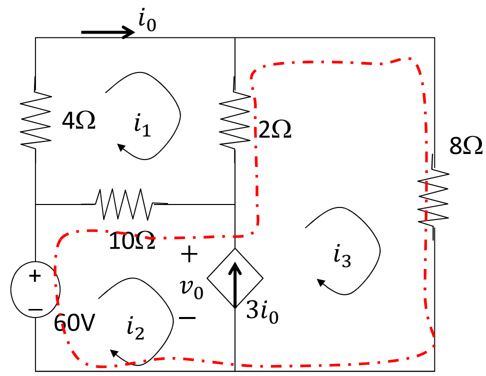
$+4i_1 + 2(i_1 - i_3) + 10(i_1 - i_2) = 0$

$16i_1 - 10i_2 - 2i_3 = 0 \quad (1)$

KVL along super mesh:

$+10(i_2 - i_1) + 2(i_3 - i_1) + 8i_3 = 60$

$-12i_1 + 10i_2 + 10i_3 = 60 \quad (2)$




Same as solution 1, the 3rd equation comes from the $3i_0$ current source:

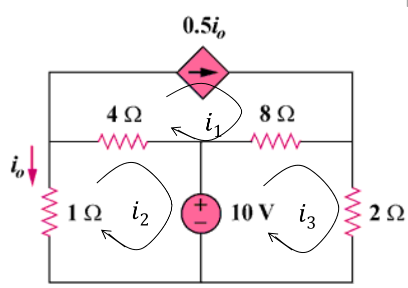
$i_3 - i_2 = 3i_0 = 3i_1$

$3i_1 + i_2 - i_3 = 0 \quad (3)$



Example: Find $i_0$ using mesh analysis

An overview:
Need 3 equations for $i_1, i_2, i_3$
One from mesh 2; (1)
One from Mesh 3; (2)
One from the dependent current source:
$i_1 = 0.5i_0 = -0.5i_2$
$i_1 + 0.5i_2 = 0 \quad (3)$




KVL along mesh 2: $i_2 + 4(i_2 - i_1) + 10 = 0$
$-4i_1 + 5i_2 = -10 \quad (1)$

KVL along mesh 3: $8(i_3 - i_1) + 2i_3 - 10 = 0$
$-8i_1 + 10i_3 = 10 \quad (2)$

All 3 equations together:

$$\begin{bmatrix} -4 & 5 & 0 \\ -8 & 0 & 10 \\ 1 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 10 \\ 0 \end{bmatrix}$$

$i_1 = 0.7143A = 5/7A$
$i_2 = -1.4286A = -10/7A$
$i_3 = 1.5714A = 11/7A$

$i_0 = -i_2 = 1.4286A$

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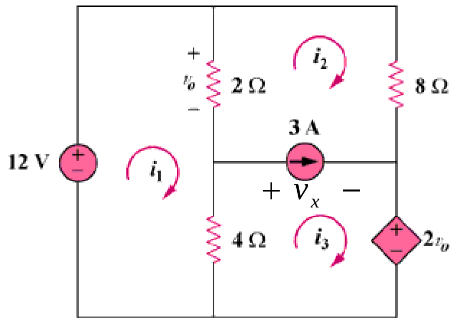
Practice 5: Find the mesh currents and $v_x$

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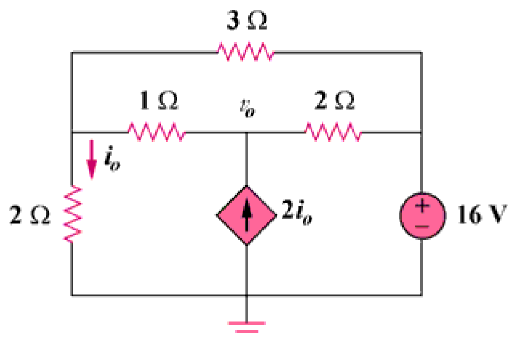
Practice 6: Find $i_0$ and $v_0$:



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Practice 7: Use mesh analysis to find $i_x$ and $v_0$: