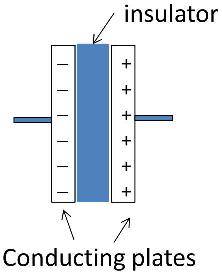
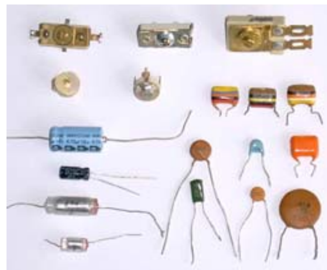
L16

Chapter 6 Capacitors and Inductors

Making preparation for dynamic circuits, which have far more applications than the static circuits we have learned so far.

§6.2 Capacitors – Store energy in electric field

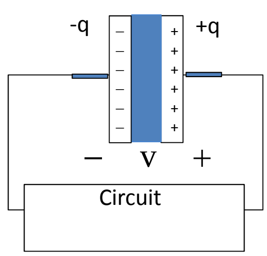




A capacitor consists of two conducting plates, separated by an insulator.

- Conduction plates: e.g., Aluminum foil
- Insulator: air, mica, ceramic, etc

L16



- No direct exchange of charge between the plates
- Electrons move from one plate to another via the rest of circuit
- Same amount of positive charge and negative charge: q

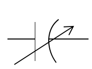
Basic Property:

The voltage v is proportional to the charge q.

The capacitance: $C \triangleq \frac{q}{v}$ Measured in Farads (F) ⇨ q= Cv, v=q/C

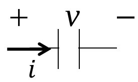
Circuit Symbol

Fixed capacitors



Variable capacitor

Capacitors are passive elements, v and i assigned by passive sign convention



Basic property: $q = Cv$

What is the relationship between v and i?
Take derivative on both side of $q = Cv$,

$$\frac{dq}{dt} = C\frac{dv}{dt} \Rightarrow i = C\frac{dv}{dt} \Leftarrow \frac{dq}{dt} = i$$

- Current is proportional to the time derivative of voltage

If current $i(t)$ is given, $dv = \frac{1}{C} i\, dt \quad \int_{t_0}^{t} dv = \int_{t_0}^{t} \frac{1}{C} i\, dt$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\, dt$$

Assume $v(-\infty) = 0$, $v(t) = \frac{1}{C}\int_{-\infty}^{t} i\, dt$

$$i = C\frac{dv}{dt} \qquad v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\, dt$$

Power delivered:

$$p = v\, i = Cv\frac{dv}{dt}$$

Energy stored:

$$w(t) = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t} Cv\frac{dv}{dt}dt$$

$$= C\int_{-\infty}^{t} v\,dv = \frac{1}{2}C(v^2(t) - v^2(-\infty)) = \frac{1}{2}Cv^2(t)$$

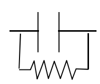
$$w = \frac{1}{2}Cv^2$$

Since v=q/C,

$$w = \frac{q^2}{2C}$$

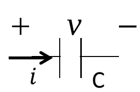
Important things to remember:

1. A capacitor is an open circuit under DC condition
   Under DC condition $v(t) = const$,
   $i = C\frac{dv}{dt} = 0$ No current through
2. The voltage on a capacitor cannot change abruptly.
   As a function of time, v(t) must be continuous
   $$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\, dt$$
   i(t) is finite, integration must be continuous
3. An ideal capacitor does not dissipate energy
4. A real, non-ideal capacitor has a large leakage resistance



e.g., 100MΩ

In summary:



$$i = C\frac{dv}{dt}; \quad v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\, dt$$

Example: Given C = 2μF, $i(t) = 6e^{-3000t}$mA, v(0)=2V, v(t)=?

In computation, everything should be in standard unit.

C = 2μF=2 $\times 10^{-6}$F, $i(t) = 6e^{-3000t} \times 10^{-3}$ A,

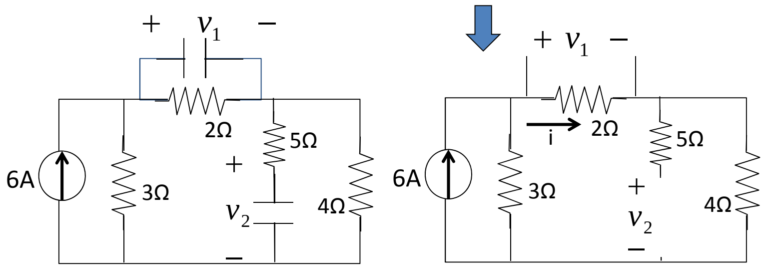
$$v(t) = v(t_0) + \frac{1}{C}\int_{0}^{t} i\, dt = 2 + \frac{1}{2\times 10^{-6}}\int_0^t 6e^{-3000t}\times 10^{-3}\, dt$$

$$= 2 + \int_0^t 3e^{-3000t}\times 10^{3}\, dt$$

$$= 2 - e^{-3000t}\Big|_0^t$$

$$= 2 - e^{-3000t} - (-1) = 3 - e^{-3000t}\text{V}$$

Example: Find v_1 and v_2 under <u>DC condition</u>

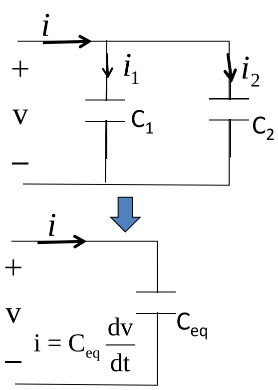


Under DC condition, capacitors are open circuit.
No current through 5Ω, no voltage drop.
v_2 same as voltage across 4Ω.
Also, 2Ω *and* 4Ω are in series.
By current division, i=6 (3/9)=2A, v_1=4V, v_2=8V

§ 6.3 Series and parallel capacitors

– Find equivalent capacitance

Parallel connection:



What is the relationship between v and i ?

By KVL, same voltage across C_1, C_2

By property of capacitor:

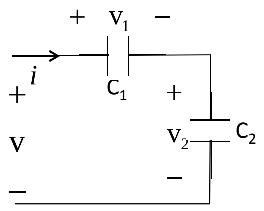
$$i_1 = C_1\frac{dv}{dt};\quad i_2 = C_2\frac{dv}{dt}$$

By KCL:

$$i = i_1 + i_2 = (C_1 + C_2)\frac{dv}{dt}$$

$\Rightarrow$ $C_{eq} = C_1 + C_2$

Series connection:



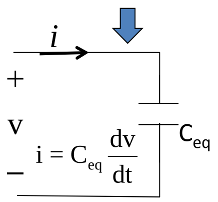
By KCL, same current through C_1, C_2
By property of capacitor:

$$v_1(t) = v_1(t_0) + \frac{1}{C_1}\int_{t_0}^{t} i\,dt$$

$$v_2(t) = v_2(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i\,dt$$

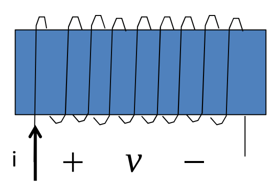
By KVL:

$$v(t) = v_1(t) + v_2(t) = v_1(t_0) + v_2(t_0) + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_{t_0}^{t} i\,dt$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

6.4 Inductors – store energy in magnetic field

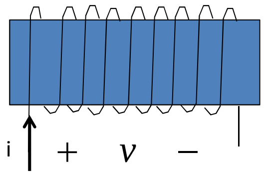


Inductors are usually constructed as a cylindrical coil
with many turns of conducting wire around a core

e.g., ferromagnetic core

Also a passive element: current i and voltage v assigned according to passive sign convention.

L16



Basic property:

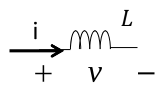
$$v = L\frac{di}{dt}$$

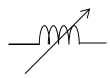
Voltage proportional to the time derivative of current

L : Inductance, measured in Henry (H)
a constant depending on the structure and material

It can be derived: $$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\,dt$$ Assume $i(-\infty) = 0$

Circuit Symbol:





Variable inductor

Energy stored: $$w = \frac{1}{2}Li^2$$

L16

Important things to remember:

1. An inductor is a short circuit under DC condition

   Under DC condition i(t)=const,

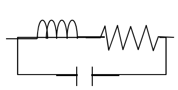
   v = $L\,di/dt = 0$ No voltage drop

2. The current on an inductor cannot change abruptly.
   As a function of time, i(t) must be continuous

   $$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\,dt$$ v(t) is finite, integration must be continuous

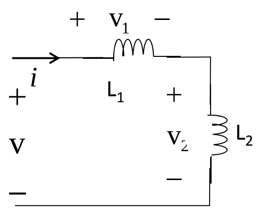
3. An ideal inductor does not dissipate energy

4. A real, non-ideal inductor



§6.5 Series and parallel Inductors

Series connection:



By KCL, same current through L_1, L_2

By property of inductor:

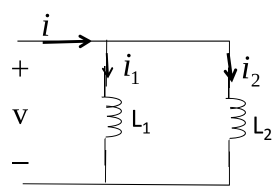
$$v_1(t)= L_1\frac{di}{dt};\quad v_2(t)= L_2\frac{di}{dt};$$

By KVL:

$$v(t)=v_1(t)+v_2(t) = (L_1 + L_2)\frac{di}{dt}$$

Equivalent inductance: $L_{eq}=L_1+ L_2$

Parallel inductors:



By KVL, same voltage across L_1, L_2

By property of inductor:

$$i_1(t) = i_1(t_0)+\frac{1}{L_1}\int_{t_0}^{t} v\,dt$$

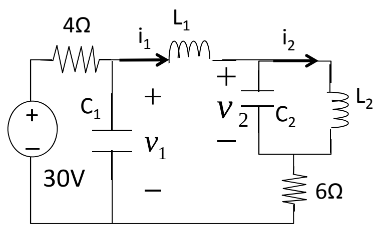
$$i_2(t_0)=i_2(t_0) +\frac{1}{L_2}\int_{t_0}^{t} v\,dt$$

By KCL:

$$\begin{aligned} i(t) &= i_1(t)+i_2(t) \\ &= i_1(t_0)+i_2(t_0)+\left(\frac{1}{L_1}+\frac{1}{L_2}\right)\int_{t_0}^{t} v\,dt \end{aligned}$$

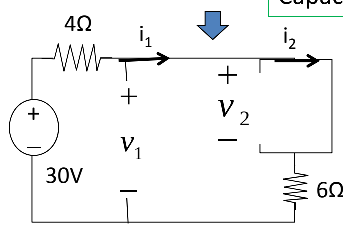
$$\frac{1}{L_{eq}}= \frac{1}{L_1}+ \frac{1}{L_2} \qquad L_{eq}= \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}}$$

Example: Find capacitor voltage and inductor current under DC condition



DC condition:

Inductor ➔ short
Capacitor ➔ open

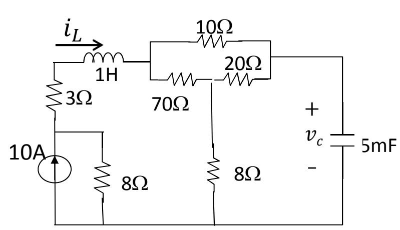


$v_2 = 0$

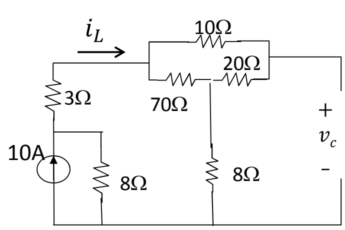
$i_1 = i_2 = 30/(4+6) = 3A$

$v_1 = 6i_2 = 18V$

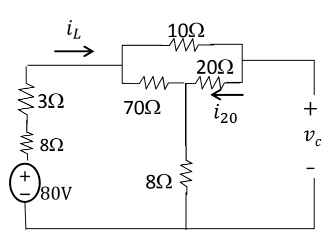
Example: Find v_c and i_L under DC condition.



Under DC condition,
Inductor = short, capacitor = open



Use source transformation:



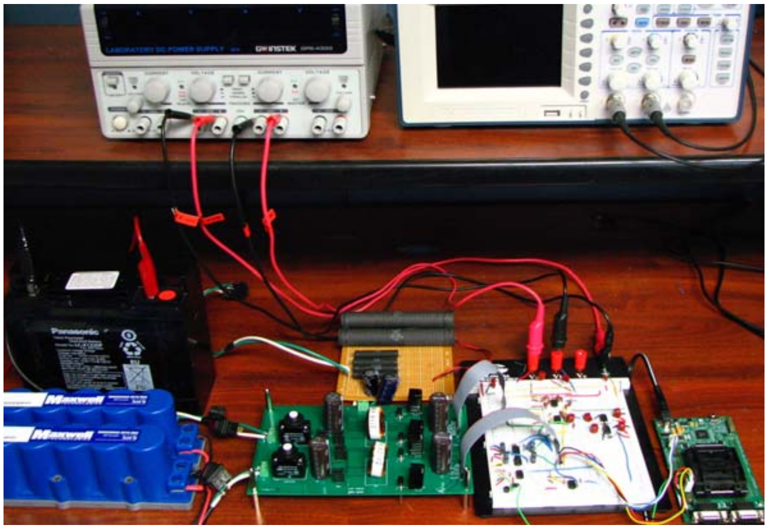
$$i_L = \frac{80}{11 + 70//30 + 8} = 2A$$

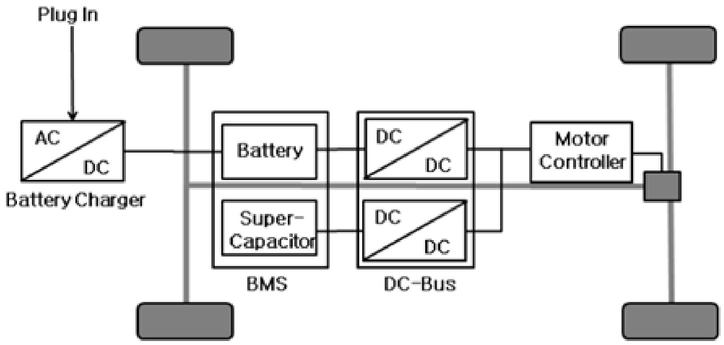
$$i_{20} = \frac{70}{100} \times 2 = 1.4A$$

$$v_c = 8i_L + 20i_{20} = 16 + 28 = 44V$$

Applications:

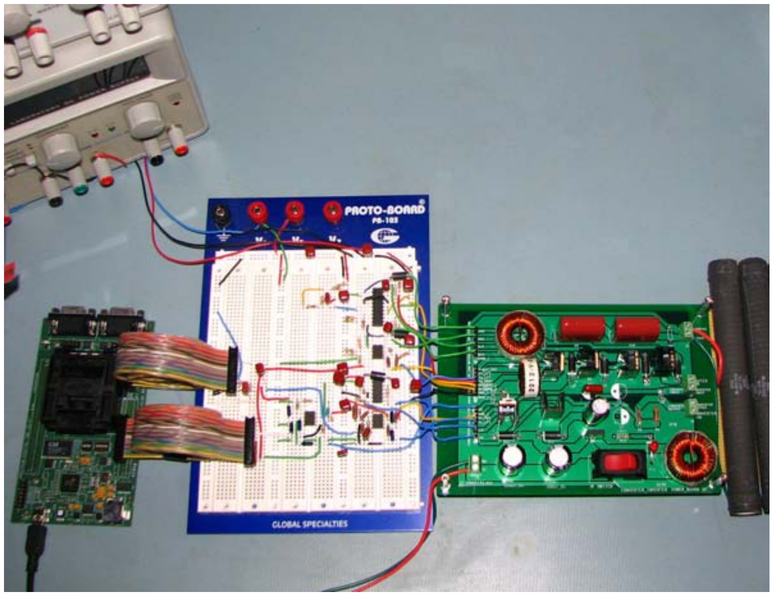
Power systems driven by battery/supercapacitor hybrid energy storage devices, by Hoeguk Jung (Ph.D student)



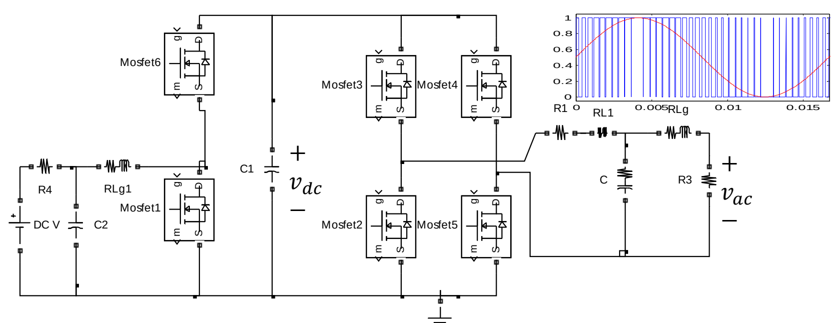


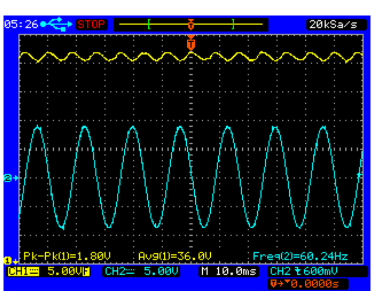
Architecture of Electric Vehicle

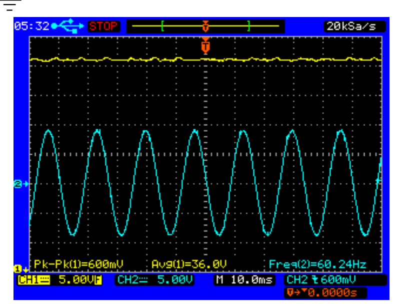
Ripple reduction in two stage inverter: experiment setup



Ripple-reduction in boost-converter-inverter







L16

Practice 1: For a capacitor with C=2F, given i(t) = 6sin4t A, v(0)=1V. Find v(t) for $t \geq 0$.

Practice 2: For an inductor with L=0.1H, given $i(t) = 10t\, e^{-5t}$ A. Find v(t), w(t).

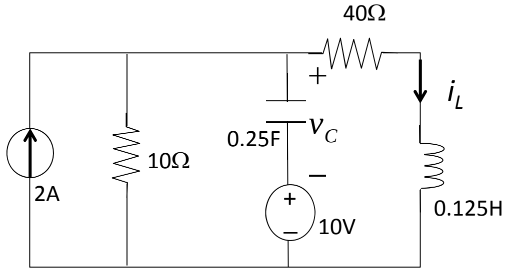
Practice 3: Given C= 4mF, i(0)=2A, and

$$v(t) = \begin{cases} 50V, & t<0 \\ Ae^{-100t} + Be^{-600t}, & t \geq 0 \end{cases}$$

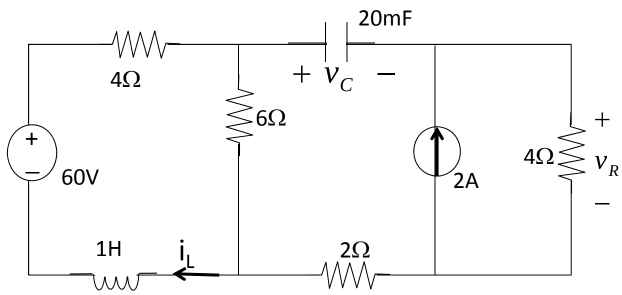
Find A and B.

L16

Practice 4: Find v_c, i_L under DC condition



Practice 5: Find i_L, v_C, v_R under DC condition.



Practice 6: Find v_C and v_R under DC condition.