Chapter 6 Capacitors and Inductors

Making preparation for dynamic circuits, which have far more applications than the static circuits we have learned so far.

§6.2 Capacitors – Store energy in electric field

A capacitor consists of two conducting plates, separated by an insulator.
- Conduction plates: e.g., Aluminum foil
- Insulator: air, mica, ceramic, etc

Basic Property:
The voltage $v$ is proportional to the charge $q$.

The capacitance: $C = \frac{q}{v}$ Measured in Farads (F) $\Rightarrow q = Cv \quad v = q/C$

Circuit Symbol
- Fixed capacitors
- Variable capacitor
Capacitors are passive elements, \( v \) and \( i \) assigned by passive sign convention

\[
\begin{array}{c}
\begin{array}{c}
+ \quad v \\
\hline
i \\
\end{array}
\end{array}
\]

Basic property: \( q = C v \)

What is the relationship between \( v \) and \( i \)?

Take derivative on both side of \( q = C v \),

\[
\frac{dq}{dt} = C \frac{dv}{dt} \quad \Rightarrow \quad i = C \frac{dv}{dt} \quad \Rightarrow \quad \frac{dq}{dt} = i
\]

- Current is proportional to the time derivative of voltage

If current \( i(t) \) is given,

\[
v(t) = v(t_0) + \frac{1}{C} \int_{t_0}^{t} i \, dt
\]

Assume \( v(-\infty) = 0 \), \( v(t) = \frac{1}{C} \int_{-\infty}^{t} i \, dt \)

Power delivered:

\[
p = v \, i = C V \frac{dv}{dt}
\]

Energy stored:

\[
w(t) = \int_{-\infty}^{t} p \, dt = \int_{-\infty}^{t} C v \frac{dv}{dt} \, dt
\]

\[
= C \int_{-\infty}^{t} v \, dv = \frac{1}{2} C (v^2(t) - v^2(-\infty)) = \frac{1}{2} C v^2(t)
\]

\[
w = \frac{1}{2} C v^2
\]

Since \( V = q/C \),

\[
w = \frac{q^2}{2C}
\]
Important things to remember:

1. A capacitor is an open circuit under DC condition

   Under DC condition \( v(t) = \text{const} \),
   \[
   i = C \frac{dv}{dt} = 0 \quad \text{No current through}
   \]

2. The voltage on a capacitor cannot change abruptly.
   As a function of time, \( v(t) \) must be continuous
   \[
   v(t) = v(t_0) + \frac{1}{C} \int_{t_0}^{t} i \, dt
   \]
   \( i(t) \) is finite, integration must be continuous

3. An ideal capacitor does not dissipate energy

4. A real, non-ideal capacitor has a large leakage resistance
   \[
   \text{e.g., } 100\Omega
   \]

In summary:

\[
\frac{v}{i} = C \frac{dv}{dt} \quad v(t) = v(t_0) + \frac{1}{C} \int_{t_0}^{t} i \, dt
\]

Example: Given \( C = 2\mu F \), \( i(t) = 6e^{-3000t} \times 10^{-3} \, A \), \( v(0) = 2V \), \( v(t) = ? \)

In computation, everything should be in standard unit.

\[
C = 2\mu F = 2 \times 10^{-6} F \] 
\( i(t) = 6e^{-3000t} \times 10^{-3} \, A, \)

\[
v(t) = v(t_0) + \frac{1}{C} \int_{0}^{t} i \, dt = 2 + \frac{1}{2 \times 10^{-6}} \int_{0}^{t} 6e^{-3000t} \times 10^{-3} \, dt
\]
\[
= 2 + \int_{0}^{t} 3e^{-3000t} \times 10^{-3} \, dt
\]
\[
= 2 - e^{-3000t} \bigg|_{0}^{t}
\]
\[
= 2 - e^{-3000t} - (-1) = 3 - e^{-3000t} V
\]
Example: Find $v_1$ and $v_2$ under DC condition

Under DC condition, capacitors are open circuit. No current through 5Ω, no voltage drop. $v_2$ same as voltage across 4Ω. Also, 2Ω and 4Ω are in series. By current division, $i=6 \times (3/9)=2A$, $v_1=4V$, $v_2=8V$

§ 6.3 Series and parallel capacitors

- Find equivalent capacitance

Parallel connection:

What is the relationship between $v$ and $i$?

By KVL, same voltage across $C_1, C_2$

By property of capacitor:

$i_1 = C_1 \frac{dv}{dt}$; $i_2 = C_2 \frac{dv}{dt}$

By KCL:

$i=i_1+i_2=(C_1+C_2)\frac{dv}{dt}$

$\Rightarrow C_{eq} = C_1 + C_2$
6.4 Inductors – store energy in magnetic field

Inductors are usually constructed as a cylindrical coil with many turns of conducting wire around a core, e.g., ferromagnetic core.

Also a passive element: current $i$ and voltage $v$ assigned according to passive sign convention.
Basic property:

\[ v = L \frac{di}{dt} \]

Voltage proportional to the time derivative of current

L : Inductance, measured in Henry (H)
A constant depending on the structure and material

It can be derived:

\[ i(t) = i(t_0) + \frac{1}{L} \int_{t_0}^{t} v dt \]

Assume \( i(\infty) = 0 \)

Circuit Symbol:

Energy stored:

\[ w = \frac{1}{2} Li^2 \]

Important things to remember:

1. An inductor is a short circuit under DC condition
   Under DC condition \( i(t) = \text{const} \),
   \[ v = L \frac{di}{dt} = 0 \quad \text{No voltage drop} \]

2. The current on an inductor cannot change abruptly.
   As a function of time, \( i(t) \) must be continuous
   \[ i(t) = i(t_0) + \frac{1}{L} \int_{t_0}^{t} v dt \quad v(t) \text{ is finite, integration must be continuous} \]

3. An ideal inductor does not dissipate energy

4. A real, non-ideal inductor
§6.5 Series and parallel Inductors

Series connection:

By KCL, same current through $L_1$, $L_2$

By property of inductor:

$$v_1(t) = L_1 \frac{di}{dt}$$
$$v_2(t) = L_2 \frac{di}{dt}$$

By KVL:

$$v(t) = v_1(t) + v_2(t) = (L_1 + L_2) \frac{di}{dt}$$

Equivalent inductance: $L_{eq} = L_1 + L_2$

Parallel inductors:

By KVL, same voltage across $L_1$, $L_2$

By property of inductor:

$$i_1(t) = i_1(t_0) + \frac{1}{L_1} \int_{t_0}^{t} v(t) dt$$
$$i_2(t) = i_2(t_0) + \frac{1}{L_2} \int_{t_0}^{t} v(t) dt$$

By KCL:

$$i(t) = i_1(t) + i_2(t)$$
$$= i_1(t_0) + i_2(t_0) + \left( \frac{1}{L_1} + \frac{1}{L_2} \right) \int_{t_0}^{t} v(t) dt$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}$
Example: Find capacitor voltage and inductor current under DC condition.

\[ v_2 = 0 \]
\[ i_1 = i_2 = \frac{30}{4+6} = 3 \text{A} \]
\[ v_1 = 6i_2 = 18 \text{V} \]

Example: Find \( v_c \) and \( i_L \) under DC condition.

Use source transformation:
Applications:

Power systems driven by battery/supercapacitor hybrid energy storage devices, by Hoeguk Jung (Ph.D student)

Architecture of Electric Vehicle
Ripple reduction in two stage inverter: experiment setup

Ripple-reduction in boost-converter-inverter
Practice 1: For a capacitor with $C=2\, \text{F}$, given $i(t) = 6\sin 4t\, \text{A}$, $v(0)=1\, \text{V}$. Find $v(t)$ for $t \geq 0$.

Practice 2: For an inductor with $L=0.1\, \text{H}$, given $i(t) = 10t\, e^{-5t}\, \text{A}$. Find $v(t)$, $w(t)$.

Practice 3: Given $C=4\, \text{mF}$, $i(0)=2\, \text{A}$, and

\[
v(t) = \begin{cases} 
50V, & t<0 \\
Ae^{-100t} + Be^{-600t}, & t \geq 0 
\end{cases}
\]

Find $A$ and $B$.

Practice 4: Find $v_c$, $i_L$ under DC condition
Practice 5: Find $i_L$, $v_C$, $v_R$ under DC condition.

Practice 6: Find $v_C$ and $v_R$ under DC condition.