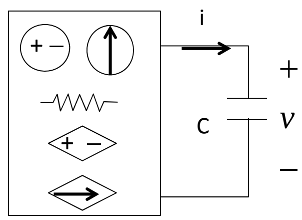
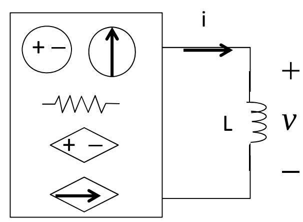
Chapter 7 First-order Circuits

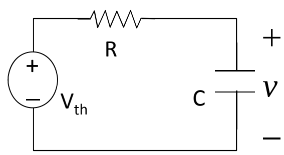
A circuit with one capacitor or inductor

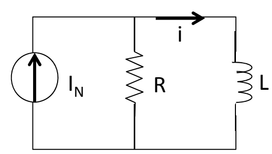




By Thevenin's theorem

By Norton's theorem



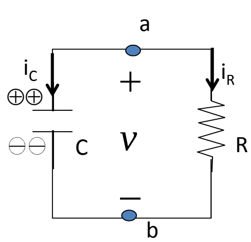


Need to know how to solve the above two simple circuits

Left : When $V_{th}= 0$, source free RC circuit ; Otherwise, step response of RC circuit

Right: When $I_N=0$, source free RL circuit Otherwise, step response of RL circuit

§ 7.2 The source free RC circuits



Given $v(0)=V_0$,

what is v(t) for t >0? $i_R(t)$?

Need to use circuit laws to derive a differential equation for v(t).
Then use math method to solve for it.

By KCL: $i_R + i_C = 0$ (1)

By property of resistor and capacitor:

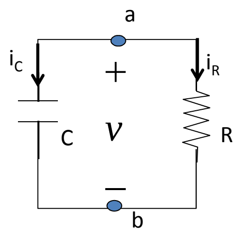
$$i_R = \frac{v}{R}; \quad i_c = C\frac{dv}{dt}$$

Plug into (1) to obtain: $\frac{v}{R} + C\frac{dv}{dt} = 0$

$$\Rightarrow \frac{dv}{dt} = -\frac{1}{RC}v \qquad (2)$$

A first order differential equation

The source free RC circuits can be described with a 1st order differential equation L17



Given $v(0)=V_0$,
what is $v(t)$ for $t > 0$? $i_R(t)$?

$$\frac{dv}{dt} = -\frac{1}{RC}v, \qquad (2)$$

Combined with initial condition $v(0)=V_0$, unique solution $v(t)$ can be found.

(2) Can be rearranged as $$\frac{dv}{v} = -\frac{1}{RC}dt \qquad (3)$$

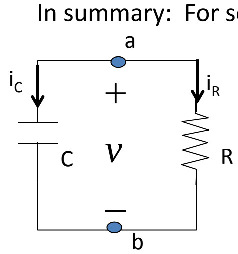
Integrating both sides $$\int_0^t \frac{dv}{v} = -\frac{1}{RC}\int_0^t dt = -\frac{t}{RC} \qquad (4)$$

Since $$\int_0^t \frac{dv}{v} = \ln v(t)\Big|_0^t = \ln v(t) - \ln v(0) = \ln\frac{v(t)}{v(0)}$$

$$\ln\frac{v(t)}{v(0)} = -\frac{t}{RC} \qquad \frac{v(t)}{v(0)} = e^{-\frac{t}{RC}} \qquad \boxed{v(t) = v(0)\, e^{-\frac{t}{RC}}} \qquad \boxed{v(t) = V_0\, e^{-\frac{t}{RC}}}$$

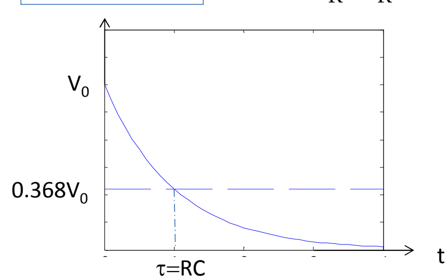
L17

In summary: For source free RC circuit,



Given $v(0)=V_0$, the solution is

$$\boxed{v(t) = V_0\, e^{-\frac{t}{RC}}} \qquad i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R}e^{-\frac{t}{RC}}$$



The natural response of the RC circuit.
It depends on:
V_0: initial condition
RC: a constant

When $t = RC$,
$$v(t) = V_0 e^{-1} = \frac{V_0}{2.7183}$$
$$= 0.368V_0$$

RC is called the <u>time constant</u>, denoted as τ: $\tau \triangleq RC$

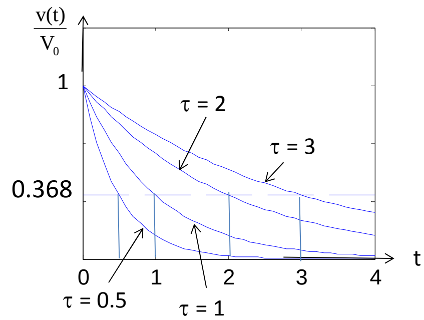
$$\boxed{v(t) = V_0\, e^{-\frac{t}{\tau}}}$$

L17

RC is called the <u>time constant</u>, denoted as τ: $\tau \triangleq RC$

$$v(t) = V_0 \, e^{-\frac{t}{\tau}}$$

At t = τ, $v(t)=V_0e^{-1}=0.368V_0$
At t= 2τ, $v(t) =V_0e^{-2}=0.13534V_0$;
At t=5τ, $v(t) = 0.00674V_0$ Capacitor almost fully discharged



Smaller τ=RC, faster decay

$$\frac{v(t)}{V_0} = e^{-\frac{t}{RC}}$$

L17

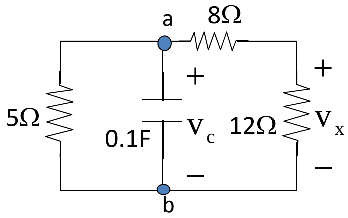
The power dissipated in R: $p = vi_R = \frac{V_0^2}{R} e^{-\frac{2t}{RC}}$

Energy absorbed by resistor from 0 to t

$$\int_0^t pdt = \int_0^t \frac{V_0^2}{R} e^{-\frac{2t}{RC}} = \frac{1}{2} C V_0^2 (1 - e^{-\frac{2t}{RC}})$$

As t $\Rightarrow \infty$, all energy stored in C will be dissipated via R.

Example: Given $v_c(0)$=15V, find $v_c(t)$, $v_x(t)$ for t > 0.

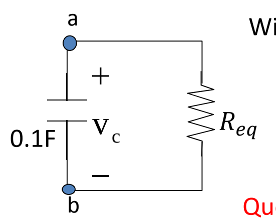


For RC circuit, always choose capacitor voltage v_c as key variable
Find $v_c(t)$ first, then express other variables in terms of it.
For this circuit,

$$v_x(t) = \frac{12}{12+8} v_c(t) = 0.6 v_c(t)$$

To find $v_c(t)$, consider the rest as a single resistor R_{eq},
We call it R_{eq} with respect to (w.r.t) the capacitor

R_{eq}=5//(8+12) = 4Ω



With $v_c(0)$ = 15V, $1/R_{eq}C$= 1/(4×0.1)=2.5

$$v(t) = v_c(0)\, e^{-\frac{t}{RC}}$$

$$v_c(t) = 15e^{-2.5t} V$$

$$v_x(t) = 0.6 \times 15e^{-2.5t} = 9e^{-2.5t} V$$

Question:
How initial condition is established?

Generally, there is a source to store energy in the capacitor for t < 0.

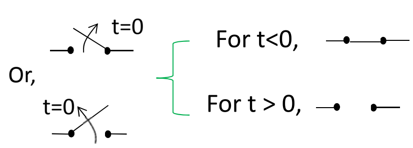
At t = 0, the source is turned off by a switch and
the circuit becomes a source free RC circuit, for t > 0.

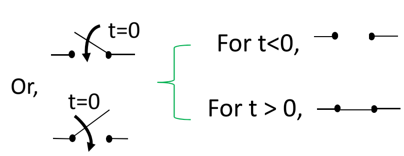
Recall two important facts about a capacitor:

1. At DC condition, a capacitor behaves like an open circuit
2. $v_c(t)$ is a continuous function of time.

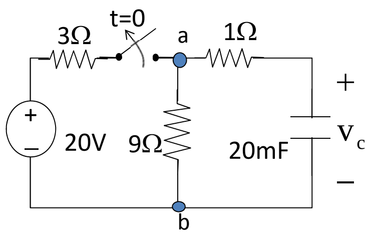
These facts will be used to determine the initial condition $v_c(0)$.

Operations of switch:





Example: The switch has been closed for a long time before it is open at t = 0. Find $v_c(t)$ for t > 0.



Interpret the condition:
Switch closed for a long time means that a DC condition has been reached at t =0^-.

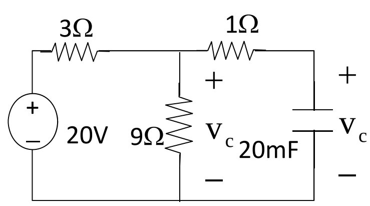
The value of v_c right before switch, $v_c(0-)$, can be found by considering the circuit before switch under DC condition.
Since $v_c(t)$ is continuous,

$$v_c(0+) = v_c(0-) = v_c(0)$$

Step 1: Find $v_c(0)$ from the DC circuit before switch (t < 0).
Step 2: Find R_{eq} with respect to C for circuit after switch (t>0)
Step 3: plug in the general formula

$$v_c(t) = v_c(0)e^{-\frac{t}{R_{eq}C}}$$

Step 1: For t < 0, or before switch.



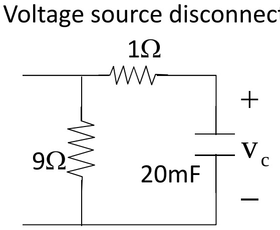
Consider the circuit under DC condition
- Capacitor behaves like an open circuit
- No current through 1Ω , thus no voltage drop
- voltage across 9 Ω = v_c

By voltage division:

$$v_c = \frac{9}{9+3} \times 20 = 15V \Rightarrow v_c(0) = 15V$$

Step 2: For t > 0, (or, after switch)

Voltage source disconnected from the right side. It has no effect to the capacitor



R_{eq}=1+9=10Ω

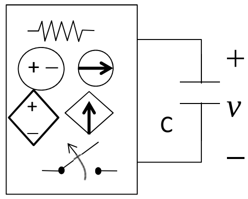
$$\frac{1}{R_{eq}C} = \frac{1}{10 \times 20 \times 10^{-3}} = 5$$

Step 3 : form $v_c(t)$

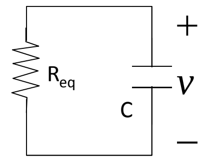
$$v_c(t) = v_c(0)e^{-\frac{t}{R_{eq}C}}$$

$$v_c(t) = 15e^{-5t}V$$

Source free RC circuit guideline



Switch turns off independent sources for t > 0.
For t > 0, by Thevenin's theorem, the two terminal circuit behaves like a single resistor R_{eq}



For t > 0,

For t > 0, $v_c(t) = v_c(0)e^{-\frac{t}{R_{eq}C}}$

Two key parameters:

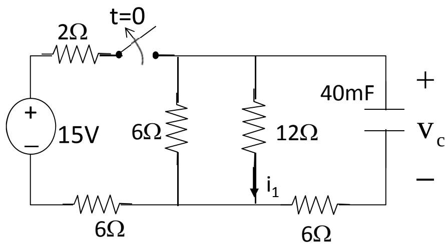
1. $v_c(0)$, initial condition for capacitor voltage.
   Established by independent sources before switch (t<0).
   Determined from the circuit before switch.
   Assume a DC condition has been reached. Capacitor behaves like open circuit
2. R_{eq}, equivalent resistance with respect to the capacitor
   for circuit after switch (t > 0).

After v(t) of capacitor is found, other variables can be obtained using basic laws

Example:

R17

Practice 1: Find $v_c(t)$ and $i_1(t)$ for t >0.



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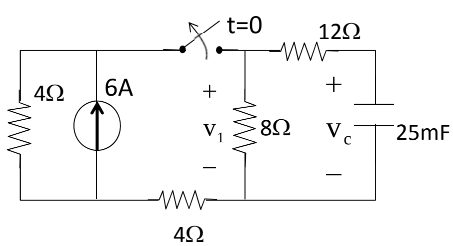
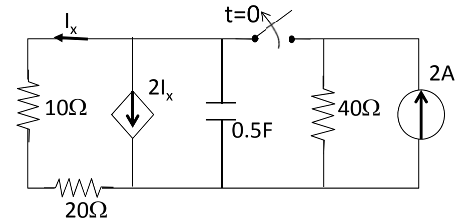
Practice 2: Find $v_c(t)$ and $v_1(t)$ for t >0.



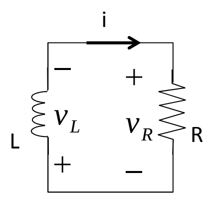
R17

Practice 3: The switch has been closed for a long time before open at t =0. Find $I_x(t)$ for t >0.



L18

7.3 The source free RL circuit



Given $i(0)=I_0$.
Need to find i(t), for all t > 0.

Assign v_L and v_R according to passive sign convention. Then

$$v_L = L\frac{di}{dt}, \quad v_R = Ri$$

By KVL, $v_L+v_R= 0$

$$\Rightarrow \quad L\frac{di}{dt} + Ri = 0$$

Compared with equation for source free RC circuit

$$\Rightarrow \quad \frac{di}{dt} = -\frac{R}{L}\,i$$

i	⇔	v
$\frac{R}{L}$	⇔	$\frac{1}{RC}$

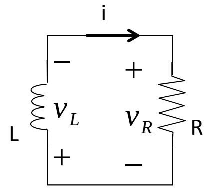
$$\frac{dv}{dt} = -\frac{1}{RC}v \qquad v(t) = v(0)e^{-\frac{t}{RC}}$$

Same math problem.

Solution: $$i(t) = i(0)e^{-\frac{Rt}{L}}$$

The source free RL circuit

L18



Given $i(0)=I_0$.

Solution:

$$i(t) = i(0)e^{-\frac{Rt}{L}}$$

Time constant:

$$\tau \triangleq \frac{L}{R}$$

$$i(t) = i(0)e^{-\frac{t}{\tau}}$$

Other variables: $v_R(t) = Ri(t) = Ri(0)e^{-\frac{Rt}{L}}$

Energy stored in inductor: $w_L(t) = \frac{1}{2}Li^2(t) = \frac{1}{2}\,i^2(0)e^{-\frac{2t}{\tau}}$

Energy consumed by resistor: $w_R(t) = \frac{1}{2}Li^2(0) - \frac{1}{2}\,i^2(0)e^{-\frac{2t}{\tau}}$

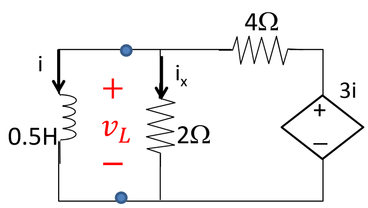
$w_R(t) + w_L(t) = \frac{1}{2}Li^2(0)$= constant

Two key parameters:

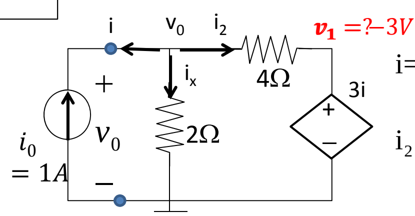
1. i(0), initial inductor current. Obtained from circuit before switch, (t<0), by treating inductor as a short circuit (under DC condition). Inductor current is continuous function of time: $i(0^-) = i(0^+) = i(0)$.
2. R, equivalent resistance with respect to inductor for circuit after switch (t > 0).

L18

Example: Given i(0)=10A, find i(t), $i_x(t)$ for t > 0.



Need to find R_{eq} with respect to inductor. Recall Thevenin's equivalent resistance from Chapter 4. Due to the dependent source, need to supply external source



$i = -1;\ i_x = \frac{v_0}{2};$

$i_2 = \frac{v_0 - 3i}{4} = \frac{v_0 + 3}{4}$

Supply i_0= 1A current,
Then $R_{eq} = v_0/1 = v_0$
v_0= ?
Use nodal analysis

KCL at v_0, $i + i_x + i_2 = 0$

$$-1 + \frac{v_0}{2} + \frac{v_0+3}{4} = 0$$

v_0= 1/3V R_{eq}=1/3 Ω

$$i(t) = i(0)e^{-\frac{R_{eq}}{L}t} = 10e^{-\frac{1/3}{0.5}t} = 10e^{-\frac{2}{3}t}A$$

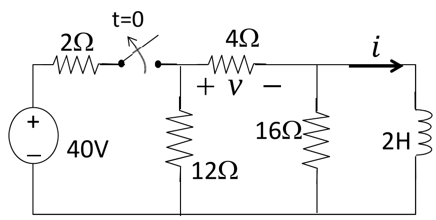
$i_x(t) = ?$

Since 2Ω in parallel with inductor:

$i_x = v_L/2 = L\frac{di}{dt}/2$

$$i_x(t) = 0.5 \times 10 \times (-\tfrac{2}{3})e^{-\frac{2}{3}t}/2 = -\tfrac{5}{3}e^{-\frac{2}{3}t}A$$

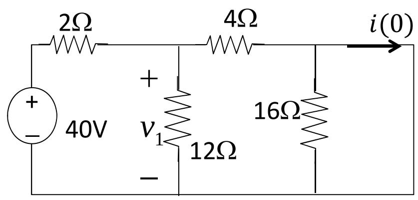
Example: The switch has been closed for a long time before open at t=0. Find $i(t), v(t)$ for t > 0.



Important facts for finding initial condition $i(0)$:

1. DC condition for t < 0
2. Under DC condition , inductor ⇔ short circuit
3. Inductor i(t) continuous $i(0-) = i(0+) = i(0)$

Step 1: Find $i(0)$ from circuit before switch (t <0).



16//0=0

No current , no voltage drop across 16Ω

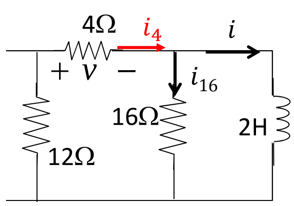
4 and 12 in parallel

Voltage division:

$$v_1 = \frac{4//12}{2+4//12} \times 40 \quad i(0) = \frac{v_1}{4} = \frac{24}{4}$$

$$= \frac{3}{2+3} \times 40 = 24V \qquad =6A$$

Step 2: For t > 0, 40V disconnected



R_{eq} with respect to inductor

R_{eq}=16//(4+12)=8Ω $\quad R_{eq}/L=4$

Step 3: $i(t) = i(0)e^{-\frac{R_{eq}}{L}t} = 6e^{-4t}A$

Step 4: $v(t) = 4(i + i_{16})$

$= 4(6-3)e^{-4t}$

$= 12e^{-4t}V$

$$i_{16}(t) = \frac{v_L}{16} = \frac{1}{16}L\frac{di}{dt}$$

$$= \frac{2}{16} \times 6 \times (-4)e^{-4t} = -3e^{-4t}A$$

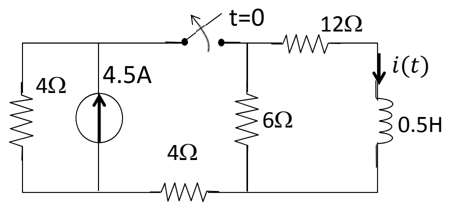
A simpler method:

$i_4(t) = \frac{1}{2}i(t)$ By current division.

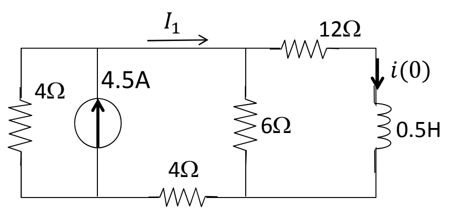
$= 3e^{-4t}A$

$v(t) = 4i_4(t) = 12e^{-4t}V$

Example: Find $i(t)$ for t >0.



Find $i(0)$ From circuit before switch:



Current division:

$I_1 = \frac{4}{4+(4+6//12)} \times 4.5 = 1.5\text{A}$

$i(0) = \frac{6}{6+12} \times 1.5 = 0.5A$

For t > 0, R_{eq} with respect to L

$R_{eq} = 6 + 12 = 18\Omega$

Put together:

$$i(t) = i(0)e^{-\frac{R_{eq}}{L}t}$$
$$= 0.5\, e^{-\frac{18}{0.5}t} = 0.5e^{-36t}A$$

§ 7.4 Singularity functions

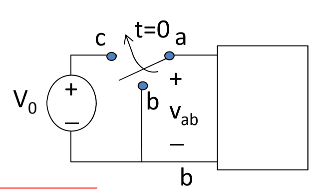
L18

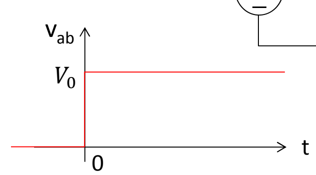
We saw switches in previous circuits.
They change the structure of the circuit, e.g., at t = 0.
At switching time t=0, some variables, such as $v_c(t)$, $i_L(t)$ are continuous, but $i_c(t)$, $v_L(t)$ may be discontinuous.

Singular functions are those having discontinuous derivative, or, being discontinuous

Step function: a basic singular function

$$v_{ab}(t) = \begin{cases} 0, & t < 0 \\ V_0, & t > 0 \end{cases}$$



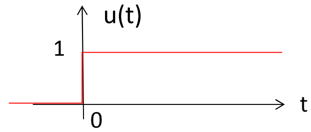


- A step function can be realized by switch, in other words,
- The operation of switch can be described by a step function

L18

The unit step function:

$$u(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$$

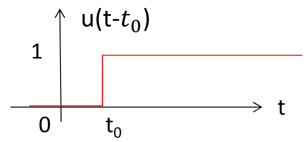


General step function can be expressed in terms of unit step:

$$v_{ab}(t)=V_0u(t)=\begin{cases}0, & t<0\\ V_0, & t>0\end{cases}$$

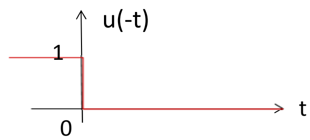
- Unit step with a time shift:

$$u(t-t_0)=\begin{cases}0, & t<t_0\\ 1, & t>t_0\end{cases}$$



- Time-reversed unit step

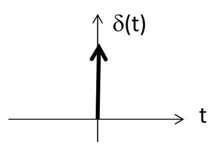
$$u(-t)=\begin{cases}1, & t<0\\ 0, & t>0\end{cases}$$



L18

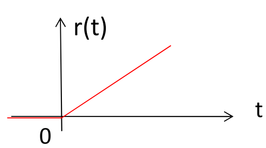
- The impulse function: the derivative of unit step

$$\delta(t)=\begin{cases}0, & t<0\\ \text{undefined}, & t=0\\ 0, & t>0\end{cases}$$



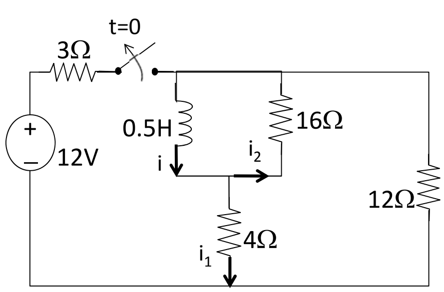
- The unit ramp function: the integration of unit step

$$r(t)=\begin{cases}0, & t<0\\ t, & t>0\end{cases}$$



Practice 4: Find $i(t)$, $i_1(t)$, $i_2(t)$ for $t > 0$.

R18

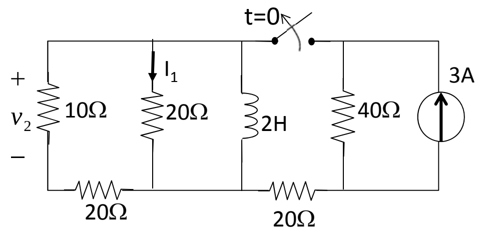


A: $i(t) = 1.5e^{-16t}\,A$

$i_1(t) = i_2(t) = 0.75e^{-16t}\,A$

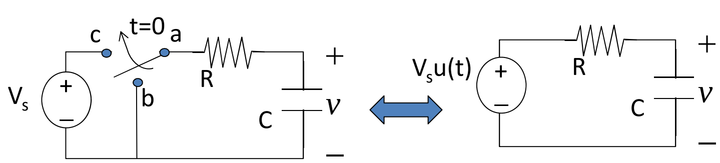
R18

Practice 5: Find $I_1(t)$ and $v_2(t)$ for t >0.



L19

§ 7.5 Step response of an RC circuit



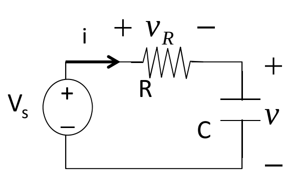
Step function:

$$u(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$$

Assume $v(0) = V_0$. Find v(t) for t > 0.

- v(t) is the response to a step function $V_s u(t)$ $\Rightarrow$ called step response.

For t > 0, $V_s u(t) = V_s$:



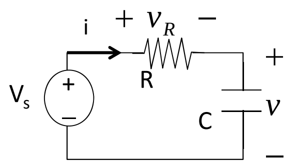
To derive a differential equation for v, assign i and v_R

$$i = C\frac{dv}{dt}; \quad v_R = Ri = RC\frac{dv}{dt}$$

By KVL, $v_R + v = V_s$

$$RC\frac{dv}{dt} + v = V_s$$ With initial condition: $v(0)=V_0$

For t > 0, $V_s u(t) = V_s$:



$$RC\frac{dv}{dt} + v = V_s$$

With initial condition: $v(0)=V_0$

$$\frac{dv}{dt} = -\frac{1}{RC}(v - V_s)$$

$$\int_0^t \frac{dv}{v-V_s} = -\frac{1}{RC}\int_0^t dt$$

$$\ln(v - V_s)|_0^t = -\frac{t}{RC}$$

$$\ln\frac{v(t) - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

$$\frac{v(t)-V_s}{V_0-V_s} = e^{-\frac{t}{RC}}$$

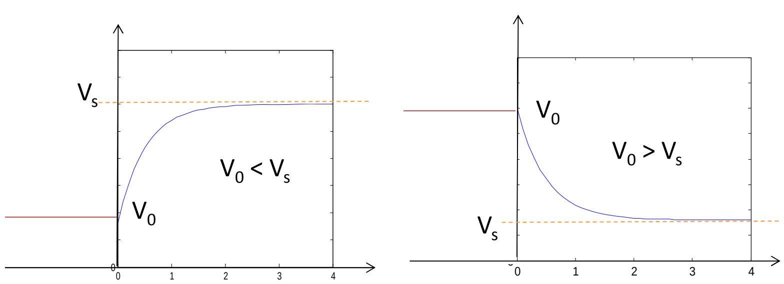
$$v(t) = V_s + (V_0 - V_s)\, e^{-\frac{t}{RC}} \quad \text{for } t > 0$$

Step response

Step response:

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0-V_s)\, e^{-\frac{t}{RC}}, & t > 0 \end{cases}$$

A step response brings a circuit from one DC condition to another DC condition



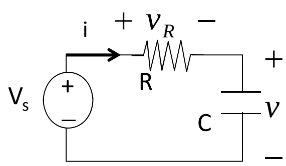
Special cases:

1. When V_s= 0, $v(t) = V_0\, e^{-\frac{t}{RC}}$ for $t > 0$ Reduce to source free circuit
2. When V_0= 0, $v(t) = V_s - V_s e^{-\frac{t}{RC}}$ for $t > 0$

L19

Step response:
$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)\, e^{-\frac{t}{RC}}, & t > 0 \end{cases}$$

You can use v(t) to find other variables:



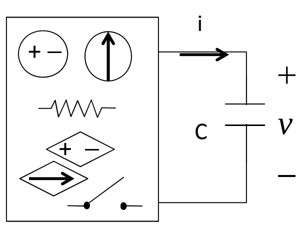
$$i(t) = C\frac{dv}{dt} = C(V_0 - V_s)\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}$$
$$= \frac{V_s - V_0}{R}e^{-\frac{t}{RC}}$$

Simpler approach by KVL:

$$i(t) = \frac{V_s - v(t)}{R} = \frac{V_s - V_0}{R}e^{-\frac{t}{RC}}$$

L19

Step response of general RC circuit

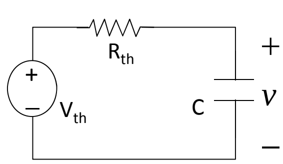


One or more switches changes structure of circuit at t = 0 (or $t = t_0$).

Problem:
need to find v(t) or, other variables for t > 0.

For t < 0, circuit behaves like a DC circuit.
Find the initial condition v(0) from this DC circuit.

For t > 0, use Thevenin's theorem:



$V_{th} \Leftrightarrow V_s$, $R_{th} \Leftrightarrow R$

$$v(t) = V_{th} + (V_0 - V_{th})\, e^{-\frac{t}{R_{th}C}}$$

Note that: $v(0) = V_{th} + (V_0 - V_{th})\, e^{-0} = V_0$

$v(\infty) = V_{th} + (V_0 - V_{th})\, e^{-\infty} = V_{th}$

Solution can also be written as

$$v(t) = v(\infty) + (v(0) - v(\infty))\, e^{-\frac{t}{R_{th}C}}$$

Solution can also be written as $$v(t) = v(\infty) + (v(0) - v(\infty))\, e^{-\frac{t}{R_{th}C}}$$

$v(\infty)$: The final value. Capacitor voltage under DC condition for circuit after $t > 0$

A short cut to form step response , – without deriving differential equation

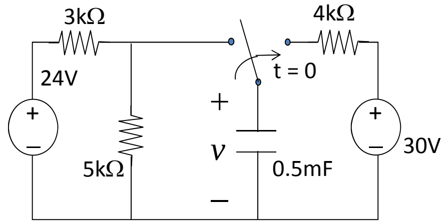
Three key parameters:

1. Initial condition $v(0)=V_0$,
   Established from circuit before switch. Obtained by solving a DC circuit, $t < 0$ (capacitor = open circuit)
2. Final condition $v(\infty)$
   Established from circuit after switch. Obtained by solving another DC circuit, $t > 0$ (capacitor = open circuit)
3. Equivalent resistance R_{th}, or R_{eq}
   with respect to capacitor for circuit after switch

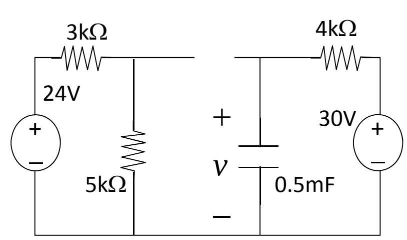
Since $v(\infty) = V_{th}$, (V_{th}, R_{th}) can be together considered as the Thevenin's equivalent, with respect to the capacitor, from circuit after switch.

The problem of finding step response breaks down to several DC circuit problems in Chapters 2 and 4.

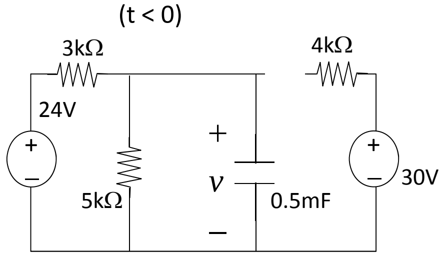
Example: Find the capacitor voltage v(t) for t > 0.



Step 2: For R_{th}, V_{th}, w.r.t capacitor, use circuit after switch



Step 1: For v(0), use circuit before switch (t < 0)



R_{th}=4kΩ; V_{th}=30V $= v(\infty)$

$$\frac{1}{R_{th}C} = \frac{1}{4\times10^{3}\times0.5\times10^{-3}} = 0.5$$

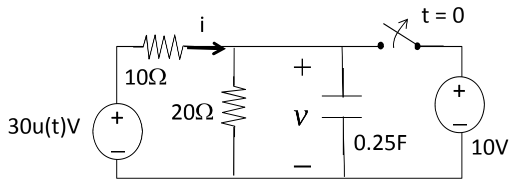
Step response:

$$v(t) = v(\infty) + \left(v(0) - v(\infty)\right)e^{-\frac{t}{R_{th}C}}$$

By voltage division, $v(0) = \frac{5}{5+3}\times 24 = 15V$

$$v(t) = 30 + (15-30)e^{-0.5t} = 30 - 15e^{-0.5t} \text{ V}$$

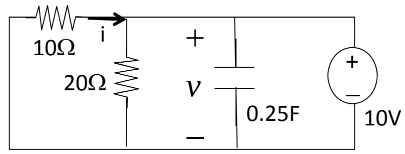
Example: Find the capacitor voltage v(t) for t > 0.



Recall:

$$30u(t)=\begin{cases}0, & t<0\\ 30, & t>0\end{cases}$$

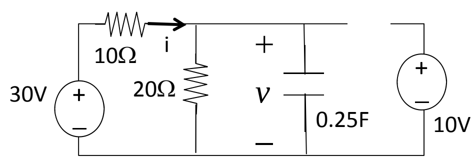
Step 1: For v(0), use circuit for t < 0



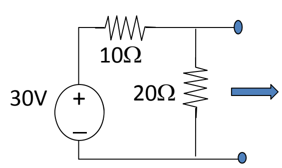
v(0)=10V

i(0-)= -1A

Step 2: For V_{th}, R_{th} , use circuit for t > 0



$$v(t)=v(\infty)+\left(v(0)-v(\infty)\right)e^{-\frac{t}{R_{th}C}}$$



V_{th}=20V=$v(\infty)$

R_{th}=10//20=20/3Ω

$$\frac{1}{R_{th}C}=\frac{1}{0.25\times 20/3}=0.6$$

$$v(t)=20+(10-20)e^{-0.6t}$$
$$=20-10e^{-0.6t}\ \text{V}$$

Step 3: How to express i(t) in terms of v(t)?

$$\text{i(t)}=\frac{30-v(t)}{10}=1+e^{-0.6t}A$$

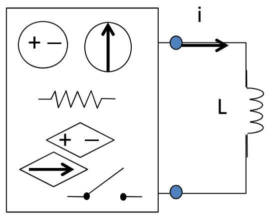
Is i(t) continuous at t = 0? i(0+)=2A i(0-)= -1A

In RC circuit, the only variable that is guaranteed to be continuous is capacitor voltage.
Other variables, such as capacitor current, resistor voltage/current can be discontinuous.
Don't make generalization like:

$$v_R(t)=v_R(\infty)+(v_R(0)-v_R(\infty))e^{-\frac{1}{R_{th}C}t}$$ (crossed out)

$$v_R(0^-)\neq v_R(0^+)$$

§ 7.6 Step response of RL circuits

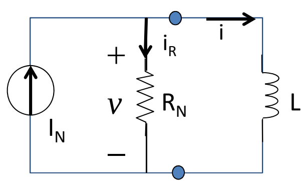


One or more switches changes structure of circuit at t = 0 (or t = t_0).

Problem: need to find i(t) or, other variables for t > 0.

For t < 0, circuit behaves like a DC circuit.
Find the initial condition i(0) from this DC circuit.

For t > 0, by Norton's theorem



By KCL, $i_R + i = I_N$

$$i_R = \frac{v}{R_N} = \frac{L}{R_N}\frac{di}{dt}$$

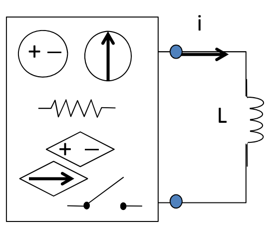
$$\frac{L}{R_N}\frac{di}{dt} + i = I_N \qquad \frac{di}{dt} = -\frac{R_N}{L}(i - I_N)$$

Same math problem as RC circuit

Step response: $i(t) = I_N + (i(0) - I_N)e^{-\frac{R_N}{L}t}$

Note that: $i(\infty) = I_N$. As $t \to \infty$, a DC condition will be reached.
$i(\infty)$ is inductor current for the DC circuit for t > 0

Step response of RL circuits



Step response:

$$i(t) = I_N + (i(0) - I_N)e^{-\frac{R_N}{L}t}$$

Since $i(\infty) = I_N$, can also be written as:

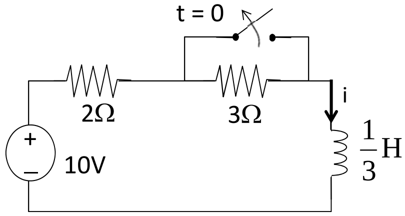
$$i(t) = i(\infty) + (i(0) - i(\infty))e^{-\frac{R_N}{L}t}$$

Three key parameters:

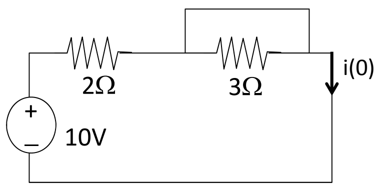
1. i(0), initial condition, from the DC circuit for t < 0.
2. $i(\infty)=I_N$, final value, from DC circuit for t > 0.
3. $R=R_N=R_{th}$, equivalent resistance w.r.t inductor from circuit for t > 0

(I_N, R_N) together as Norton's equivalent, w.r.t inductor, for t > 0.

Example: Find i(t) for t > 0.

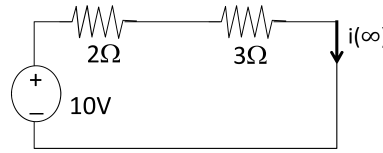


Step 1: find i(0). Consider DC circuit for t < 0.



i(0) = 10/2=5A

Step 2: Find i(∞) = I_N from DC circuit for t > 0.



i(∞) = 10/5=2A

R_N w.r.t. L for t > 0:

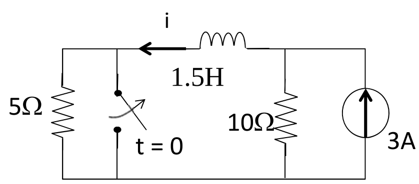
R_N= 2+3=5Ω $\quad \frac{R_N}{L} = \frac{5}{1/3} = 15$

Step response:

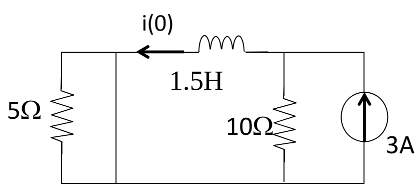
$$i(t) = i(\infty) + \big(i(0) - i(\infty)\big)e^{-\frac{R_N t}{L}}$$

$$i(t) = 2 + (5-2)e^{-15t}A$$
$$= 2 + 3e^{-15t}A$$

Example: Find i(t) for t > 0.

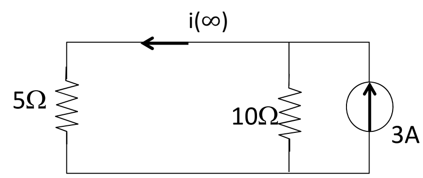


Step 1:
Find i(0). Consider DC circuit (t < 0)



i(0)=3A

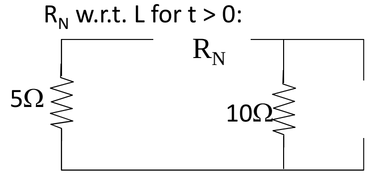
Step 2: Find i(∞) = I_N from DC circuit (t > 0)



By current division:

$$i(\infty) = \frac{10}{10+5} \times 3 = 2A$$

R_N w.r.t. L for t > 0:



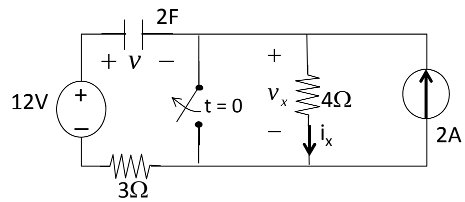
R_N=15Ω, R_N/L=10

$$i(t) = i(\infty) + \big(i(0) - i(\infty)\big)e^{-\frac{R_N t}{L}}$$

$$i(t) = 2 + (3-2)e^{-10t}A$$
$$= 2 + e^{-10t}A$$

R19

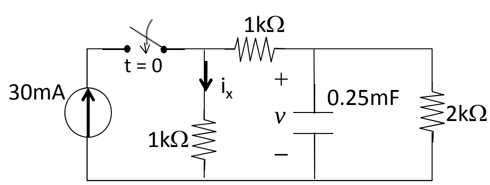
Practice 6: Find v(t), i_x(t) for t > 0.



A: $v(t) = 4 + 8e^{-t/14} V$
$i_x(t) = 2 - (8/7)e^{-t/14} A$

R19

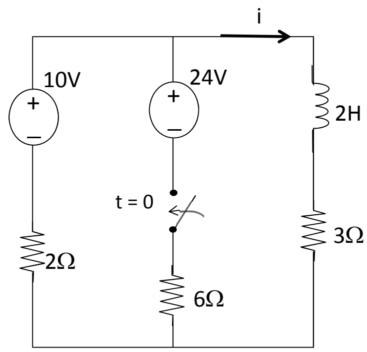
Practice 7: Find i_x(t) for t > 0.



A: $v(t) = 15(1 - e^{-4t}) V$
$i_x(t) = 22.5 \times 10^{-3} - 7.5 \times 10^{-3} e^{-4t} A$

Practice 8: Find i(t) for all t .

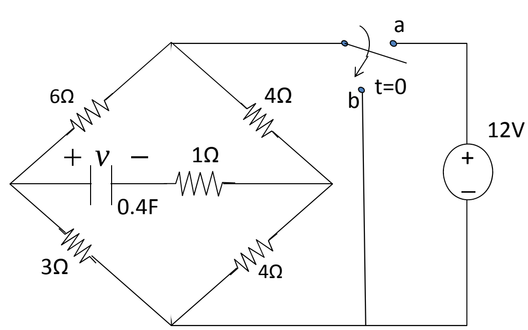
R19



A: $i(0) = 2A, i(\infty) = 3A, R_N = 4.5\Omega$
$i(t) = 3 - e^{-2.25t} A$

Practice 9: Switch connected to “a” for a long time before connected to “b” at t=0. Find v(t) for t > 0.

R19



A: $v(t) = -2e^{-t/2}V$