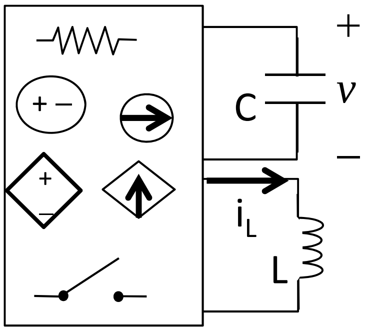
Step response of RC circuit: $v(t) = v(\infty) + \big(v(0) - v(\infty)\big)\ e^{-\frac{1}{R_{th}C}t}$

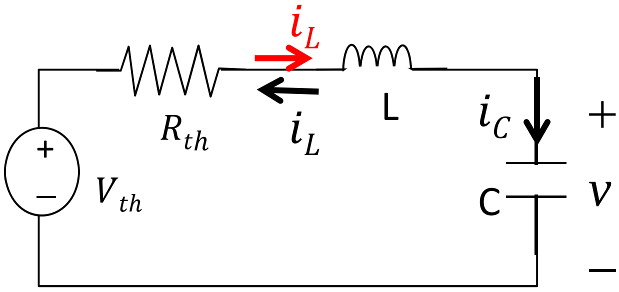
Step response of RL circuit: $i(t) = i(\infty) + \big(i(0) - i(\infty)\big)\ e^{-\frac{R_{th}}{L}t}$

Key points for step resp. of series RLC circuits:



Obtain v(0), $i_L(0)$ from circuit before switch

For circuit after switch (t> 0), obtain Thevenin's equivalent (V_{th}, R_{th}) w.r.t LC



Assign i_C by passive sign convention

$$i_C(0^+) = \pm i_L(0)$$

Depending on how i_L is assigned. Then $\dfrac{dv(0^+)}{dt} = \dfrac{i_C(0^+)}{C}$

For t > 0, $\dfrac{d^2v}{dt^2} + \dfrac{R_{th}}{L}\dfrac{dv}{dt} + \dfrac{1}{LC}v = \dfrac{V_{th}}{LC}$;

$\alpha = \dfrac{R_{th}}{2L}$, $\omega_0 = \dfrac{1}{\sqrt{LC}}$, $v(\infty) = V_{th}$

Key parameters to obtain:

- $v(0), i_L(0)$ from circuit before switch
- V_{th}, R_{th} from circuit after switch.

Then follow straightforward math:

Case 1: $\alpha > \omega_0$; $s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$, $v(t) = v(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Find A_1,A_2 from
$$\begin{cases} v(0) = v(\infty) + A_1 + A_2 \\ \dfrac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2 \end{cases}$$

Case 2: $\alpha = \omega_0$; $s_1 = s_2 = -\alpha$, $v(t) = v(\infty) + (A_1 + A_2 t)e^{-\alpha t}$

Find A_1,A_2 from
$$\begin{cases} v(0) = v(\infty) + A_1 \\ \dfrac{dv(0^+)}{dt} = -\alpha A_1 + A_2 \end{cases}$$

Case 3: $\alpha < \omega_0$; $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, $s_1, s_2 = -\alpha \pm j\omega_d$, $v(t) = v(\infty) + e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$

Find B_1,B_2 from
$$\begin{cases} v(0) = v(\infty) + B_1 \\ \dfrac{dv(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 \end{cases}$$