Problem 1

(a) Since inductor current and capacitor voltage are continuous, to find $i(0^+)$ and $v(0^+)$, consider the circuit before switch.
Under DC condition, inductor short, capacitor open:

$$i(0^+) = i(0^-) = \frac{18}{6} A = 3A, \quad v(0^+) = v(0^-) = 18V$$

(b) To find $\frac{dv(0^+)}{dt}$, consider circuit at $t = 0^+$ (After switch)

$$i_c(0^+) = -i(0^+) = -3A, \quad \frac{dv(0^+)}{dt} = \frac{ic}{C} = -\frac{3}{0.4} = -7.5V/s$$

(c) To find $i(\infty)$ and $v(\infty)$, consider circuit after switch under DC condition. Since no power supply, $i(\infty) = 0, v(\infty) = 0$.

Problem 2

$$\alpha = \omega_0 = 5, \quad b = 0, \quad \Rightarrow A_\infty = 0$$

Thus $i(t) = (A_1 + A_2 t)e^{-5t}$

From initial condition:

$$i(0) = A_1 = 3$$

$$i'(0) = -5A_1 + A_2 = 1 \Rightarrow A_2 = 1 + 5A_1 = 16$$

$$\Rightarrow \quad i(t) = (3 + 16t)e^{-5t}A$$

Problem 3

$$\alpha = 2, \omega_0 = 2\sqrt{2}, \quad b = 48, \quad \text{Since } \omega_0 > \alpha, \text{ use solution for case 3}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 2, \quad A_\infty = \frac{48}{8} = 6$$

The general solution: $v(t) = 6 + e^{-2t}(Acos2t + Bsin2t)$

From initial condition:

$$v(0) = 6 + A = 4 \quad \Rightarrow \quad A = -2$$

$$v'(0) = -2e^{-2t}(Acos2t + Bsin2t) + e^{-2t}(-2Acos2t + 2Bcos2t)|_{t=0}$$

$$= -2A + 2B = -20 \quad \Rightarrow \quad B = -10 + A = -12$$

$$v(t) = 6 + e^{-2t}(-2cos2t - 12sin2t)$$
Problem 4

Before switch

15A

5Ω

0.25H

10Ω

0.04F

Step 1: Find initial condition from circuit before switch.
Under DC condition: \( v(0) = 5 \times 15 = 75V, \ i(0) = 0A \)

Step 2: Find \( \frac{dv(0^+)}{dt} \) and \( v(\infty) \) From circuit after switch

At \( t = 0^+ \), \( i_c(0^+) = i(0) = 0A, \)
\[ \frac{dv(0^+)}{dt} = 0 \text{ V/s} \]
At \( t = \infty \), No power supply, \( v(\infty) = 0V \)

With respect to LC, \( R = 10\Omega \)

Step 3: Math computation
\[
\alpha = \frac{R}{2L} = \frac{10}{0.5} = 20, \ \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{0.1} = 10, \ v(\infty) = 0
\]
Since \( \alpha > \omega_0 \), case 1.
\[
s_1, s_2 = -20 \pm \sqrt{400 - 100} = -20 \pm 10\sqrt{3} = -2.679, -37.32
\]
The general solution:
\[
v(t) = Ae^{-2.679t} + Be^{-37.32t}
\]
From initial condition:
\[
v(0) = A + B = 75
v'(0) = -2.679A - 37.32B = 0
\]
\[
\begin{align*}
A &= 80.801 \\
B &= -5.801
\end{align*}
\]
\[
v(t) = 80.801e^{-2.679t} - 5.801e^{-37.32t}V
\]
Problem 5

It is more convenient to find capacitor voltage first. Then take derivative to find loop current. Assign capacitor voltage as in figures.

Step 1: find \(v(0), \ i(0)\) from circuit for \(t < 0\). Under DC condition:
\[
 i(0) = \frac{12}{6} = 2A, \ v(0) = -2A \times 2\Omega = -4V \quad \text{(capacitor and 2\Omega in parallel)}
\]

Step 2: At \(t = 0^+\), capacitor and inductor in series, \(i_c(0^+) = i(0) = 2A, \)
\[
\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{c} = \frac{2}{0.25} = 8V/s
\]
At \(t = \infty\), 12V not connected to LC, \(v(\infty) = 0\)
With respect to LC, \(R = 2\Omega\)

Step 3: Math computation
\[
\alpha = \frac{R}{2L} = \frac{2}{1} = 2, \quad \omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{\frac{1}{2\sqrt{2}}} = 2\sqrt{2},
\]
Since \(\alpha < \omega_0\), use Case 3
\[
\omega_d = \sqrt{8 - 4} = 2,
\]
General solution: \(v(t) = e^{-2t}(A\cos 2t + B\sin 2t)\)
From initial condition:
\[
v(0) = A = -4
\]
\[
v'(0) = -2A + 2B = 8 \quad \Rightarrow \quad B = 4 + A = 0
\]
Thus \(v(t) = -4e^{-2t}\cos 2t V\)
Since \(i(t)\) and \(i_c(t)\) are the same,
\[
 i(t) = C \frac{dv}{dt} = 0.25(-4(-2)e^{-2t}\cos 2t + 8e^{-2t}\sin 2t)
 = 2e^{-2t}(\cos 2t + \sin 2t)A
\]
Step 1: Find \( v(0) \) and inductor current \( i(0) \) from circuit for \( t < 0 \).
Since no voltage across 6Ω, \( v(0) \) is the same as voltage across 60Ω, which is in parallel with 15Ω + 25Ω, \( 60/(15+25) \Omega = 24 \Omega \)
By voltage division,
\[
v(0) = 40 \times \frac{24}{36+24} = 16V, \quad \text{And} \quad i(0) = 0.
\]

Step 2: At \( t = 0^+ \), \( v'(0^+) = \frac{i(0)}{C} = 0 \)
At \( t = \infty \), since no power supply, \( v(\infty) = 0 \)
With respect to LC, \( R = 6 + 60|| (15 + 25) = 30 \Omega \)

Step 3: Math
\[
\alpha = \frac{R}{2L} = \frac{30}{6} = 5, \quad \omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{3} = 3
\]
Since \( \alpha > \omega_0 \), case 1,
\[
s_1, s_2 = -5 \pm \sqrt{25 - 9} = -9, -1
\]
The general solution: \( v(t) = Ae^{-\alpha t} + Be^{-\omega_0 t} \)
From initial condition:
\[
v(0) = A + B = 16 \quad \Rightarrow \quad \begin{cases} A = -2 \\ B = 18 \end{cases}
\]
\[
v'(0) = -9A - B = 0
\]
\[
v(t) = -2e^{-\alpha t} + 18e^{-t} V
\]
Problem 7

**Step 1:** For \( t < 0 \), \( 4u(-t) = 4A \), current source on; \( 60u(t) = 0V \), voltage source off (short).
\[
v(0) = -4 \times 6V = -24V, \quad i(0) = 0, \quad v'(0) = 0.
\]

**Step 2:** For \( t > 0 \), \( 4u(-t) = 0 \), current source off (open), \( 60u(t)=60V \), voltage source on,
At \( t = \infty \), \( v(\infty) = 60V \),
\( R \) with respect to \( LC \), \( R = 4 + 2 = 6\Omega \)

**Step 3:**
\[
\alpha = \frac{R}{2L} = \frac{6}{2} = 3, \quad \omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{0.2} = 5,
\]
\( w_0 > \alpha \), Case 3
\( \omega_d = \sqrt{25 - 9} = 4 \),
The general solution: \( v(t) = 60 + e^{-3t}(Acos4t + Bsin4t) \)

From initial condition:
\[
v(0) = 60 + A = -24 \quad \Rightarrow \quad A = -84
\]
\[
v'(0) = -3A + 4B = 0 \quad \Rightarrow \quad B = 0.75A = -63
\]
\[
v(t) = 60 + e^{-3t}(-84cos4t - 63sin4t)V
\]
Problem 8

Step 1: For \( t < 0 \), \( 3u(t) = 0A \), right side source off (open)

\( v(0) = 6 \times (10||5) = 6 \times \frac{5 \times 10}{5 + 10} = 20V \),

\( i(0) = \frac{v(0)}{5} = \frac{20}{5} = 4A \),

Step 2: For \( t > 0 \), \( i \) and \( i_c \) are opposite, \( i_c(0^+) = -i(0) = -4A \)

\( v'(0) = -\frac{i(0)}{c} = -1V/s \)

At \( t = \infty \), \( v(\infty) = 3 \times 5V = 15V \),

\( R = 5\Omega \)

Step 3:

\[
\alpha = \frac{R}{2L} = \frac{5}{2} = 2.5, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{2} = 0.5
\]

Case 1: \( s_1, s_2 = -2.5 \pm \sqrt{6.25 - 0.25} = -4.949, -0.0505 \)

General solution:

\( v(t) = 15 + A e^{-4.949t} + Be^{-0.0505t} \)

Using initial condition:

\( v(0) = 15 + A + B = 20 \)

\( v'(0) = -4.949A - 0.0505B = -1 \)

\[
\begin{align*}
A &= 0.1526 \\
B &= 4.8474
\end{align*}
\]

\( v(t) = 15 + 0.1526e^{-4.949t} + 4.8474e^{-0.0505t} \ V \)
Problem 9

For $t < 0$, 2$I(t) = 0A$, current source off (open circuit)

$v(0) = 30V, \quad i(0) = 0A$

For $t > 0$, \( v'(0^+) = \frac{i(0)}{C} = 0V/s \).

At $t = \infty$, \( i = 0, v_5 = 2 \times 5 = 10V \)

By KVL, \( v(\infty) = 30 + v_5 = 40V \),

\[ R = 2 + 1 + 5 = 8\Omega \]

General solution:

\[ v(t) = 40 + e^{-0.8t}(Acos0.6t + Bsin0.6t) \]

Compute A and B from initial condition:

\[ v(0) = 40 + A = 30 \quad \Rightarrow \quad A = -10 \]

\[ v'(0) = -0.8A + 0.6B = 0 \quad \Rightarrow \quad B = -13.33 \]

\[ v(t) = 40 - e^{-0.8t}(10cos0.6t + 13.33sin0.6t) \] V

\[ i(t) = \frac{d}{dt} v = 0.2(0.8e^{-0.8t}(10cos0.6t + 13.33sin0.6t)) - e^{-0.8t}(-6sin0.6t + 8cos0.6t)) \]

\[ = 3.33e^{-0.8t}sin0.6tA \]
Problem 10

For $t < 0$, $v(0) = 0$, since no current to charge the capacitor

By current division, $i(0) = \frac{6}{3+6} \times 3 = 1.8A$

For $t > 0$, $v'(0) = \frac{i(0)}{C} = 1.8/0.02 = 90V/s$,

At $t = \infty$, $v(\infty) = 3 \times 6 - 12 = 6V$

With respect to LC, $R = 6 + 14 = 20\Omega$

Assign capacitor voltage $v$.

For $t < 0$, $v(0) = 0$, since no current to charge the capacitor

By current division, $i(0) = \frac{6}{3+6} \times 3 = 1.8A$

For $t > 0$, $v'(0) = \frac{i(0)}{C} = 1.8/0.02 = 90V/s$,

At $t = \infty$, $v(\infty) = 3 \times 6 - 12 = 6V$

With respect to LC, $R = 6 + 14 = 20\Omega$

$$\alpha = \frac{R}{2L} = \frac{14+6}{4} = 5,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{0.2} = 5, \quad \alpha = \omega_0, \text{ case 2}$$

General solution:
$$v(t) = 6 + (A + Bt)e^{-5t}$$

$v(0) = 6 + A = 0 \quad \Rightarrow A = -6$

$v'(0) = -5A + B = 90 \quad \Rightarrow B = 90 + 5A = 60$

$$v(t) = 6 + (-6 + 60t)e^{-5t}$$

$$i(t) = C \frac{dv}{dt} = 0.02(-5(-6 + 60t)e^{-5t} + 60e^{-5t})$$

$$= (1.8 - 6t)e^{-5t}A$$
Problem 11

For $t < 0$, $v_2 = 4 \times 2 = 8V,$
$v_6 = 6 \times 3 = 18V$

By KVL,
$v(0) = v_2 - v_6 = 8 - 18$
$v(0) = -10V$
$i(0) = 0A$

For $t > 0$,

At $t = 0^+$, $v'(0^+) = \frac{i(0)}{C} = 0$

At $t = \infty$,
$v(\infty) = -3 \times 6 = -18V$

(No voltage across $2\Omega$ due to short circuit)
$R = 6\Omega$

\[\alpha = \frac{R}{2L} = \frac{6}{2} = 3, \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{0.2} = 5,\]

$\omega_0 > \alpha, \quad \text{Case 3}$

\[\omega_d = \sqrt{25 - 9} = 4\]

General solution:
$v(t) = -18 + e^{-3t}(A\cos 4t + B\sin 4t)$

To satisfy initial condition:
$v(0) = -18 + A = -10 \quad A = 8$
$v'(0) = -3A + 4B = 0 \quad B = 6$

$v(t) = -18 + e^{-3t}(8\cos 4t + 6\sin 4t)V$