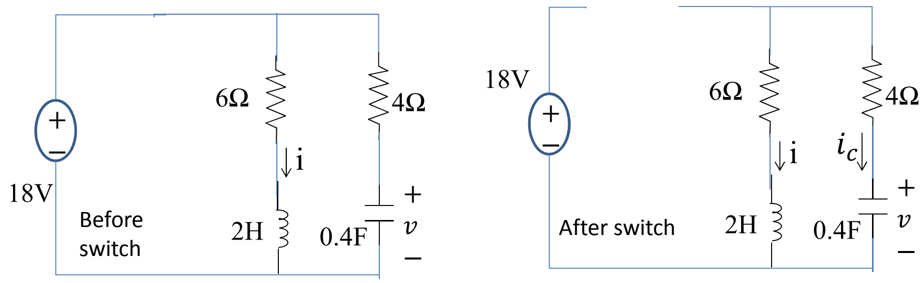
Problem 1



(a) Since inductor current and capacitor voltage are continuous, to find $i(0^+)$ and $v(0^+)$, consider the circuit before switch. Under DC condition, inductor short, capacitor open:
$i(0^+) = i(0^-) = 18/6\ A = 3A$, $v(0^+) = v(0^-) = 18V$

(b) To find $\frac{dv(0^+)}{dt}$, consider circuit at $t = 0^+$ (After switch)
$i_c(0^+) = -i(0^+) = -3A$, $\frac{dv(0^+)}{dt} = \frac{i_C}{C} = -\frac{3}{0.4} = -7.5V/s$

(c) To find $i(\infty)$ and $v(\infty)$, consider circuit after switch under DC condition. Since no power supply, $i(\infty) = 0$, $v(\infty) = 0$.

Problem 2

$\alpha = \omega_0 = 5, \quad b = 0, \quad \Rightarrow A_\infty = 0$
Thus $i(t) = (A_1 + A_2 t)e^{-5t}$
From initial condition:
$i(0) = A_1 = 3$
$i'(0) = -5A_1 + A_2 = 1 \Rightarrow A_2 = 1 + 5A_1 = 16$
$\Rightarrow i(t) = (3 + 16t)e^{-5t} A$

Problem 3

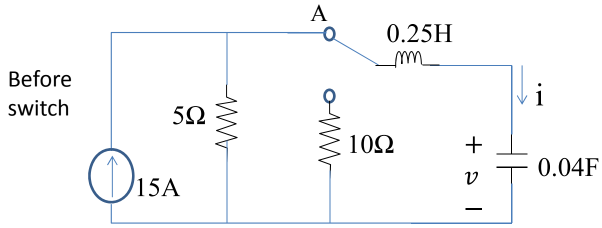
$\alpha = 2, \omega_0 = 2\sqrt{2}, \ b = 48, \ Since\ \omega_0 > \alpha,$ use solution for case 3

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 2, \ A_\infty = \frac{48}{8} = 6$

The general solution: $v(t) = 6 + e^{-2t}(Acos2t + Bsin2t)$
From initial condition:
$v(0) = 6 + A = 4 \Rightarrow A = -2$
$v'(0) = -2e^{-2t}(Acos2t + Bsin2t) + e^{-2t}(-2Asin2t + 2Bcos2t)|_{t=0}$
$= -2A + 2B = -20 \Rightarrow B = -10 + A = -12$

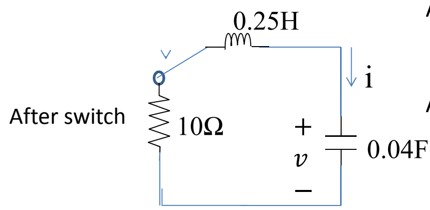
$$v(t) = 6 + e^{-2t}(-2cos2t - 12sin2t)$$

Problem 4



Step 1: Find initial condition from circuit before switch.
Under DC condition: $v(0) = 5 \times 15 = 75V,\ i(0) = 0\text{A}$

Step 2: Find $\frac{dv(0^+)}{dt}$ $and\ v(\infty)$ From circuit after switch



At $t = 0^+,\ i_c(0^+) = i(0) = 0A,$

$$\frac{dv(0^+)}{de} = 0\ V/s$$

At $t = \infty$, No power supply, $v(\infty) = 0V$
With respect to LC, $R = 10\Omega$

Step 3: Math computation

$$\alpha = \frac{R}{2L} = \frac{10}{0.5} = 20, \omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{0.1} = 10, v(\infty) = 0$$

Since $\alpha > \omega_0$, case 1.

$$s_1, s_2 = -20 \pm \sqrt{400 - 100} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$

The general solution:

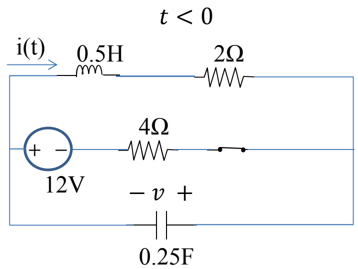
$$v(t) = Ae^{-2.679t} + Be^{-37.32t}$$

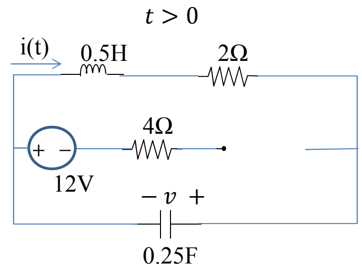
From initial condition:

$v(0) = A + B = 75$
$v'(0) = -2.679A - 37.32B = 0$ ⇒ $\begin{cases} A = 80.801 \\ B = -5.801 \end{cases}$

$$v(t) = 80.801e^{-2.679t} - 5.801e^{-37.32t}V$$

Problem 5





It is more convenient to find capacitor voltage first. Then take derivative to find loop current. Assign capacitor voltage as in figures.

Step 1: find $v(0)$, $i(0)$ from circuit for $t < 0$. Under DC condition:

$i(0) = \frac{12}{6} = 2A$, $v(0) = -2A \times 2\Omega = -4V$ (capacitor and 2Ω in parallel)

Step 2: At $t = 0^+$, capacitor and inductor in series, $i_c(0^+) = i(0) = 2A$,

$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{2}{0.25} = 8V/s$

At $t = \infty$, 12V not connected to LC, $v(\infty) = 0$

With respect to LC, $R = 2\Omega$

Step 3: Math computation

$$\alpha = \frac{R}{2L} = \frac{2}{1} = 2, \qquad \omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{\frac{1}{2\sqrt{2}}} = 2\sqrt{2},$$

Since $\alpha < \omega_0$, use Case 3

$\omega_d = \sqrt{8-4} = 2,$

General solution: $v(t) = e^{-2t}(A\cos 2t + B\sin 2t)$

From initial condition:

$v(0) = A = -4$

$v'(0) = -2A + 2B = 8$ ⇒ $B = 4 + A = 0$

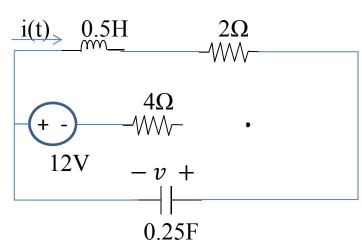
Thus $v(t) = -4e^{-2t}\cos 2t\ V$

Since $i(t)$ and $i_c(t)$ are the same,

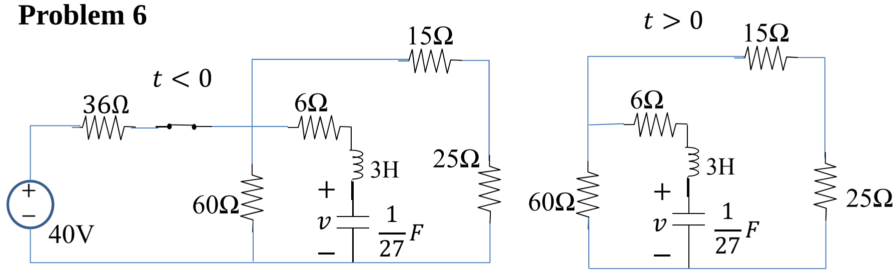
$i(t) = C\frac{dv}{dt} = 0.25(-4(-2)e^{-2t}\cos t2t$

$+8e^{-2t}\sin 2t)$

$= 2e^{-2t}(\cos 2t + \sin 2t)A$



Problem 6



Step 1: Find $v(0)$ *and inductor current* $i(0)$ from circuit for t < 0.
Since no voltage across 6Ω, v(0) is the same as voltage across 60Ω, which is in parallel with 15Ω + 25Ω, 60//(15+25)Ω=24Ω
By voltage division,

$v(0) = 40 \times \frac{24}{36+24} = 16V$, And $i(0) = 0$.

Step 2: At $t = 0^+$, $v'(0^+) = \frac{i(0)}{C} = 0$

At $t = \infty$, since no power supply, $v(\infty) = 0$

With respect to LC, $R = 6 + 60||(15 + 25) = 30\Omega$

Step 3: Math

$$\alpha = \frac{R}{2L} = \frac{30}{6} = 5, \qquad \omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{\frac{1}{3}} = 3,$$

Since $\alpha > \omega_0$, case 1,

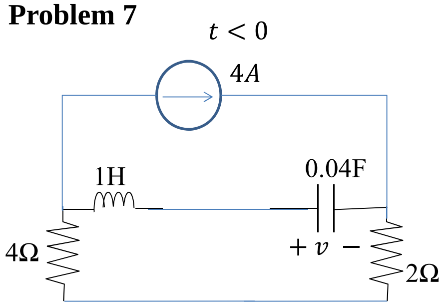
$$s_1, s_2 = -5 \pm \sqrt{25 - 9} = -9, -1$$

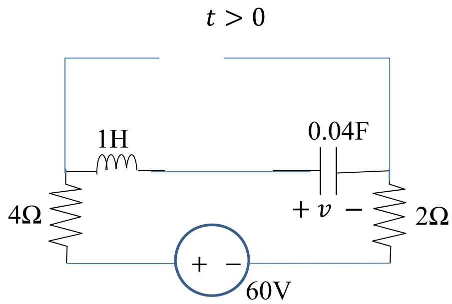
The general solution: $v(t) = Ae^{-9t} + Be^{-t}$
From initial condition:

$v(0) = A + B = 16$
$v'(0) = -9A - B = 0$
$\Rightarrow \begin{cases} A = -2 \\ B = 18 \end{cases}$

$$v(t) = -2e^{-9t} + 18e^{-t}\ V$$

Problem 7





Step 1: For t < 0, 4u(-t) = 4A, current source on;
60u(t) = 0V , voltage source off (short).
$v(0) = -4 \times 6V = -24V, \ \ i(0) = 0, \ v'(0) = 0.$

Step 2: For t > 0, 4u(-t) = 0, current source off (open), 60u(t)=60V, voltage source on,
At $t = \infty$, $v(\infty) = 60V$,
R with respect to LC, $R = 4 + 2 = 6\Omega$

Step 3:

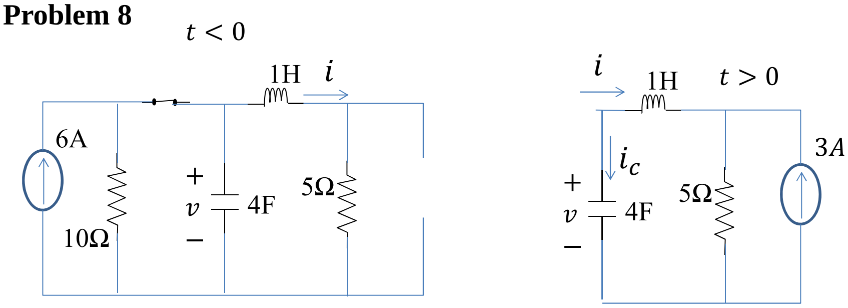
$$\alpha = \frac{R}{2L} = \frac{6}{2} = 3, \omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{0.2} = 5,$$

$w_0 > \alpha$, Case 3
$\omega_d = \sqrt{25 - 9} = 4,$
The general solution: $v(t) = 60 + e^{-3t}(Acos4t + Bsin4t)$

From initial condition :
$v(0) = 60 + A = -24$ ⇒ A = -84
$v'(0) = -3A + 4B = 0$ ⇒ B = 0.75A = -63

$v(t) = 60 + e^{-3t}(-84cos4t - 63sin4t)$V

Problem 8



Step 1: For t < 0, $3u(t) = 0A$, right side source off (open)

$v(0) = 6 \times (10||5) = 6 \times \frac{5\times 10}{5+10} = 20V$,

$i(0) = \frac{v(0)}{5} = \frac{20}{5} = 4A$,

Step 2: For t > 0, i and i_c are opposite , $i_c(0^+) = -i(0) = -4A$

$v'(0) = -\frac{i(0)}{C} = -1\text{V/s}$

At $t = \infty$, $v(\infty) = 3 \times 5V = 15V$,

$R = 5\Omega$

Step 3:

$$\alpha = \frac{R}{2L} = \frac{5}{2} = 2.5,\ \omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{2} = 0.5$$

Case 1: $s_1, s_2 = -2.5 \pm \sqrt{6.25 - 0.25} = -4.949, -0.0505$

General solution:

$$v(t) = 15 + Ae^{-4.949t} + Be^{-0.0505t}$$

Using initial condition:

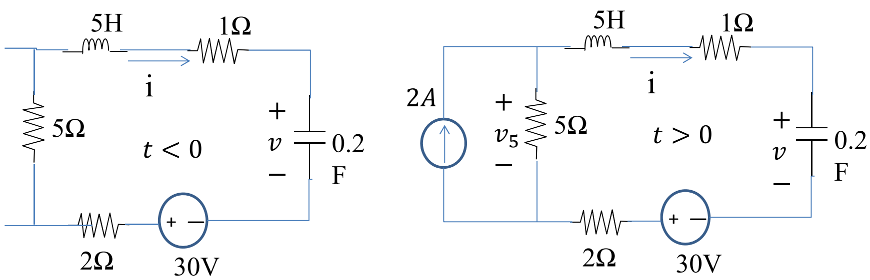
$v(0) = 15 + A + B = 20$

$v'(0) = -4.949A - 0.0505B = -1$

$\Rightarrow \begin{cases} A = 0.1526 \\ B = 4.8474 \end{cases}$

$v(t) = 15 + 0.1526e^{-4.949t} + 4.8474e^{-0.0505t}\text{V}$

Problem 9



For t < 0, 2u(t)=0A, current source off (open circuit)

$v(0) = 30V, \quad i(0) = 0A$

For t > 0, $v'(0^+) = \frac{i(0)}{C} = 0V/s.$

At $t = \infty$, $i = 0$, $v_5 = 2 \times 5 = 10V$,

By KVL, $v(\infty) = 30 + v_5 = 40V,$

R=2+1+5=8Ω

$\alpha = \frac{R}{2L} = \frac{8}{10} = 0.8, \quad \omega_0 = \frac{1}{\sqrt{CL}} = 1,$

$\alpha < \omega_0$, Case 3, $\omega_d = \sqrt{1 - 0.64} = 0.6$

General solution:

$$v(t) = 40 + e^{-0.8t}(Acos0.6t + Bsin0.6t)$$

Compute A and B from initial condition:

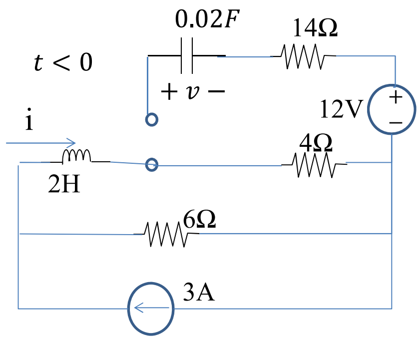
$v(0) = 40 + A = 30 \quad \Rightarrow \quad$ A = -10

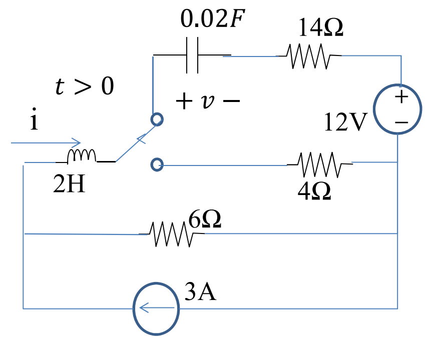
$v'(0) = -0.8A + 0.6B = 0 \quad \Rightarrow \quad$ B = -13.33

$v(t) = 40 - e^{-0.8t}(10cos0.6t + 13.33sin0.6t)$ V

$$i(t) = C\frac{dv}{dt} = 0.2(0.8e^{-0.8t}(10cos0.6t + 13.33sin0.6t)$$
$$-e^{-0.8t}(-6sin0.6t + 8cos0.6t))$$
$$= 3.33e^{-0.8t}sin0.6t\text{A}$$

Problem 10





Assign capacitor voltage v.

For t < 0, $v(0) = 0$, since no current to charge the capacitor

By current division, $i(0) = \frac{6}{4+6} \times 3 = 1.8A$

For t > 0, $v'(0^+) = \frac{i(0)}{C} = 1.8/0.02 = 90V/s$,

At $t = \infty$, $v(\infty) = 3 \times 6 - 12 = 6V$

With respect to LC, $R = 6 + 14 = 20\Omega$

$\alpha = \frac{R}{2L} = \frac{14+6}{4} = 5,$

$\omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{0.2} = 5, \quad \alpha = \omega_0,$ case 2

General solution:

$v(t) = 6 + (A + Bt)e^{-5t}$

$v(0) = 6 + A = 0$ ⟹ A = -6

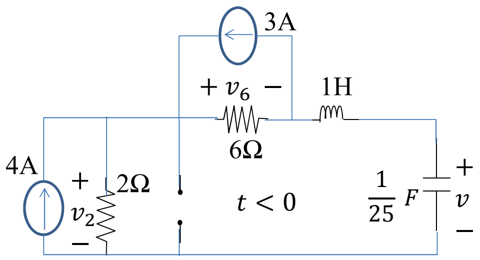
$v'(0) = -5A + B = 90$ ⟹ B = 90+5A = 60

$$v(t) = 6 + (-6 + 60t)e^{-5t}$$

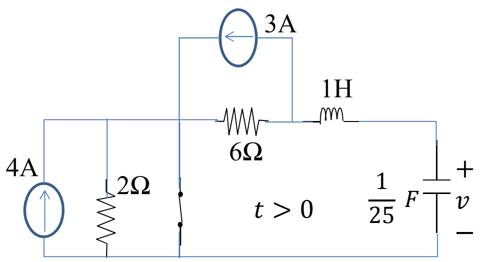
$$i(t) = C\frac{dv}{dt} = 0.02(-5(-6 + 60t)e^{-5t} + 60e^{-5t})$$

$$= (1.8 - 6t)e^{-5t}A$$

Problem 11



For $t < 0$, $v_2 = 4 \times 2 = 8V$,
$v_6 = 6 \times 3 = 18V$
By KVL,
$v(0) = v_2 - v_6 = 8 - 18$
$= -10V$
$i(0) = 0A$



For t > 0,
At t = 0^+, $v'(0^+) = \frac{i(0)}{C} = 0$
At $t = \infty$,
$v(\infty) = -3 \times 6 = -18V$
(No voltage across 2Ω due to short circuit)
$R = 6\Omega$

$\alpha = \frac{R}{2L} = \frac{6}{2} = 3$, $\omega_0 = \frac{1}{\sqrt{CL}} = \frac{1}{0.2} = 5$,
$w_0 > \alpha$, $Case\ 3$
$\omega_d = \sqrt{25 - 9} = 4$
General solution:
$v(t) = -18 + e^{-3t}(Acos4t + Bsin4t)$

To satisfy initial condition:
$v(0) = -18 + A = -10$ A = 8
$v'(0) = -3A + 4B = 0$ B = 6

$v(t) = -18 + e^{-3t}(8cos4t + 6sin4t)$V