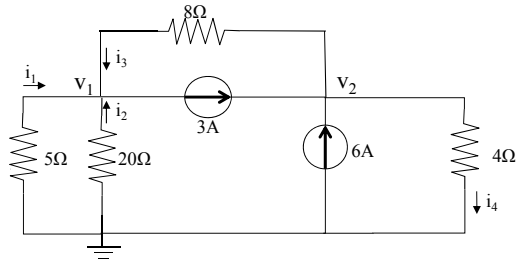
Homework #4 solution

Problem 1 solution



KCL at v_1: $i_1 + i_2 + i_3 = 3;$

at v_2: $-i_3 - i_4 + 3 + 6 = 0;\ \Rightarrow\ i_3 + i_4 = 9$

Express resistor currents:

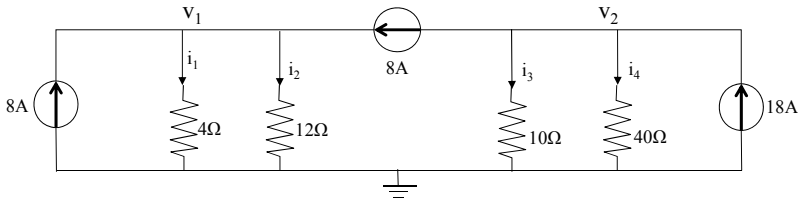
$$i_1 = -\frac{v_1}{5};\ i_2 = -\frac{v_1}{20};\ i_3 = \frac{v_2 - v_1}{8};\ i_4 = \frac{v_2}{4}$$

Plug into KCL equations:

$$-\frac{v_1}{5} - \frac{v_1}{20} + \frac{v_2 - v_1}{8} = 3;$$
$$\frac{v_2 - v_1}{8} + \frac{v_2}{4} = 9;$$
$$\Rightarrow \quad -3v_1 + v_2 = 24;$$
$$-v_1 + 3v_2 = 72$$

$$v_1 = 0V$$
$$v_2 = 24V$$

Problem 2 solution



KCL at v_1, v_2: $i_1 + i_2 = 8 + 8 = 16;$

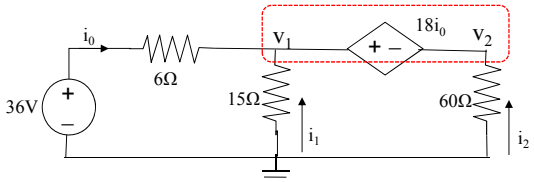
$i_3 + i_4 = 18 - 8 = 10;$

Resistor currents: $i_1 = \frac{v_1}{4}; i_2 = \frac{v_1}{12}; i_3 = \frac{v_2}{10}; i_4 = \frac{v_2}{40}$

$$\frac{v_1}{4} + \frac{v_1}{12} = 16;$$
$$\frac{v_2}{10} + \frac{v_2}{40} = 10$$

$$v_1 = 48V; v_2 = 80V;$$
$$i_1 = 12A; i_2 = 4A;$$
$$i_3 = 8A; i_4 = 2A$$

Problem 3 solution



Due to the floating dependent voltage source, a super-node need to be formed

KCL at super-node: $i_0 + i_1 + i_2 = 0$

By the dependent voltage source: $v_1 - v_2 = 18i_0$

Resistor currents: $i_0 = \frac{36 - v_1}{6}, \quad i_1 = -\frac{v_1}{15}, \quad i_2 = -\frac{v_2}{60}$

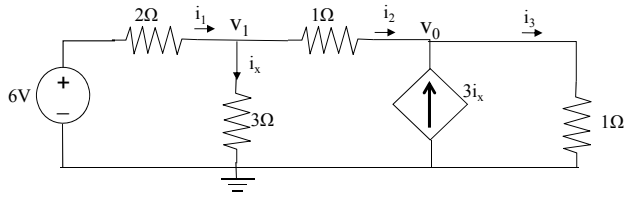
Plug in:

$$\begin{cases} \frac{36 - v_1}{6} - \frac{v_1}{15} - \frac{v_2}{60} = 0 \\ v_1 - v_2 = 18 \cdot \frac{36 - v_1}{6} \end{cases} \Rightarrow \begin{cases} 14v_1 + v_2 = 360 \\ 4v_1 - v_2 = 108 \end{cases} \Rightarrow \begin{cases} v_1 = 26V \\ v_2 = -4V \end{cases}$$

$$i_0 = \frac{36 - 26}{6} A = 1.667A$$

$i_0 = 1.667A$

Problem 4 solution



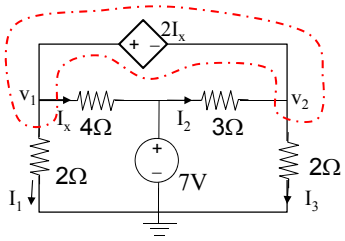
$i_1 = i_x + i_2$

$i_2 + 3i_x = i_3$

$i_1 = \frac{6 - v_1}{2}, \quad i_2 = \frac{v_1 - v_0}{1}, \quad i_3 = \frac{v_0}{1}, \quad i_x = \frac{v_1}{3}$

$$\begin{cases} \frac{6 - v_1}{2} = \frac{v_1}{3} + \frac{v_1 - v_0}{1} \\ \frac{v_1 - v_0}{1} + v_1 = v_0 \end{cases} \Rightarrow \begin{cases} v_1 = 3.6V \\ v_0 = 3.6V \end{cases}$$

Problem 5 solution



$$I_1 = \frac{v_1}{2};\quad I_2 = \frac{7 - v_2}{3};$$
$$I_3 = \frac{v_2}{2};\quad I_x = \frac{v_1 - 7}{4}$$

Combine v_1 and v_2 into a super node:

At super node: $I_1+I_x-I_2+I_3=0$

$$\frac{v_1}{2} + \frac{v_1 - 7}{4} - \frac{7 - v_2}{3} + \frac{v_2}{2} = 0, \qquad \times 12$$

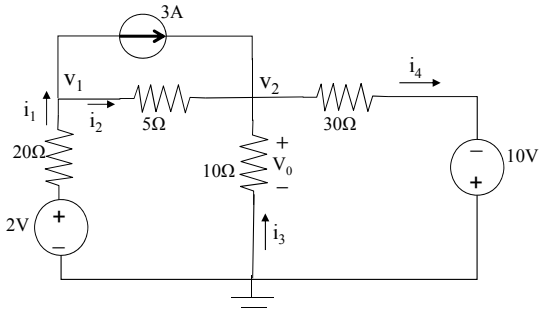
$$\Rightarrow \quad 6v_1 + 3v_1 - 21 - 28 + 4v_2 + 6v_2 = 0$$
$$9v_1 + 10v_2 = 49 \qquad (1)$$

By the dependent voltage source $2I_x$: $v_1-v_2= 2I_x$

$$\Rightarrow \quad v_1 - v_2 = 2 \times \frac{v_1 - 7}{4}$$
$$2v_1 - 2v_2 = v_1 - 7$$
$$v_1 - 2v_2 = -7 \qquad (2)$$

Solving (1) and (2):
$v_1= 1V$;
$v_2= 4V$;
$I_x=(v_1-7)/4= -1.5A$

Problem 6 solution

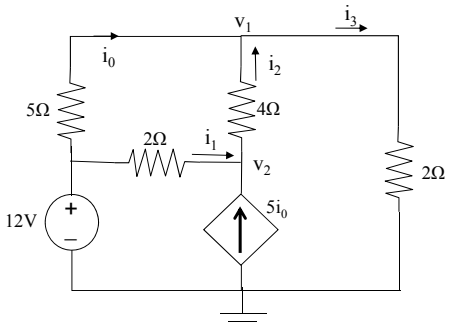


$$i_1 = i_2 + 3$$
$$i_2 + 3 + i_3 = i_4$$
$$i_1 = \frac{2 - v_1}{20},\quad i_2 = \frac{v_1 - v_2}{5},\quad i_3 = -\frac{v_0}{10},\quad i_4 = \frac{v_2 + 10}{30}$$

$$\begin{cases} \dfrac{2 - v_1}{20} = \dfrac{v_1 - v_2}{5} + 3 \\ \dfrac{v_1 - v_2}{5} + 3 - \dfrac{v_0}{10} = \dfrac{v_2 + 10}{30} \\ v_2 = v_0 \end{cases} \Rightarrow \begin{cases} 5v_1 - 4v_2 = -58 \\ 3v_1 - 5v_2 = -40 \\ v_0 = v_2 \end{cases} \Rightarrow \begin{cases} v_1 = -10V \\ v_2 = 2V \\ v_0 = 2V \end{cases}$$

$v_1 = -10V;$
$v_2 = 2V;$
$v_0 = 2V$

Problem 7 solution



$$i_0 + i_2 = i_3$$

$$i_1 + 5i_0 = i_2$$

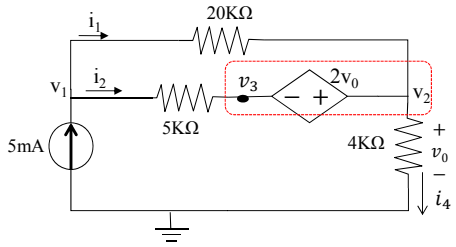
$$i_0 = \frac{12 - v_1}{5}, \quad i_1 = \frac{12 - v_2}{2}, \quad i_2 = \frac{v_2 - v_1}{4}, \quad i_3 = \frac{v_1}{2}$$

$$\begin{cases} \frac{12 - v_1}{5} + \frac{v_2 - v_1}{4} = \frac{v_1}{2} \\ \frac{12 - v_2}{2} + 12 - v_1 = \frac{v_2 - v_1}{4} \end{cases} \Rightarrow \begin{cases} 19v_1 - 5v_2 = 48 \\ v_1 + v_2 = 24 \end{cases} \Rightarrow \begin{cases} v_1 = 7V \\ v_2 = 17V \end{cases}$$

$$i_0 = \frac{12 - v_1}{5} = 1A$$

$v_1 = 7V;$
$v_2 = 17V;$
$i_0 = 1A$

Problem 8 solution



There is an additional voltage v_3 which is different from v_1, v_2
Due to the floating voltage source, form a super-node

$Resistor\ voltages$: $i_1 = \frac{v_1 - v_2}{20K}$; $i_2 = \frac{v_1 - v_3}{5K}$; $i_4 = \frac{v_2}{4K}$

$KCL\ at\ v_1$: $i_1 + i_2 = 5m;$

KCL $at\ Supernode$: $i_1 + i_2 - i_4 = 0$

By the voltage source: $v_2 - v_3 = 2v_0 = 2v_2$

Put current expressions into KCL equations:

$$\frac{v_1 - v_2}{20K} + \frac{v_1 - v_3}{5K} = 5m;$$

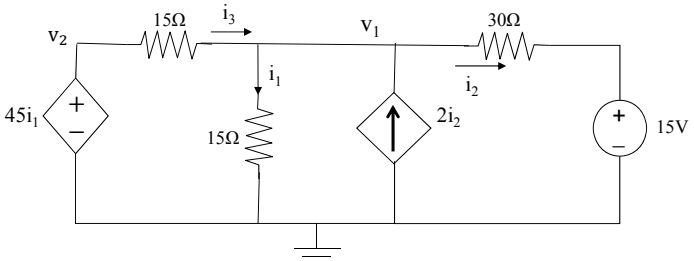
$$\frac{v_1 - v_2}{20K} + \frac{v_1 - v_3}{5K} - \frac{v_2}{4K} = 0;$$

$$v_2 - v_3 = 2v_2$$

$$5v_1 - v_2 - 4v_3 = 100;$$
$$5v_1 - 6v_2 - 4v_3 = 0;$$
$$v_2 + v_3 = 0$$

$v_1 = 8V; v_2 = 20V, v_3 = -20V$

Problem 9 solution



Assign v_2, Then $v_2 = 45i_1$ (1)

Resistor currents: $i_1 = \frac{v_1}{15}$; $i_2 = \frac{v_1-15}{30}$; $i_3 = \frac{v_2-v_1}{15}$

KCL at v_1: $i_1 + i_2 - 2i_2 - i_3 = 0 \Rightarrow i_1 - i_2 - i_3 = 0$

Put in current expressions: $\frac{v_1}{15} - \frac{v_1-15}{30} - \frac{v_2-v_1}{15} = 0$

Simplify to obtain: $3v_1 - 2v_2 = -15$; (2)

From (1): $v_2 = 45\frac{v_1}{15} = 3v_1$ (3)

Solving (2) and (3) : $v_1 = 5V$; $v_2 = 15V$;

$$i_1 = \frac{v_1}{15} = \frac{5}{15} = 0.333A$$
$$i_2 = \frac{v_1 - 15}{30} = -0.333A$$