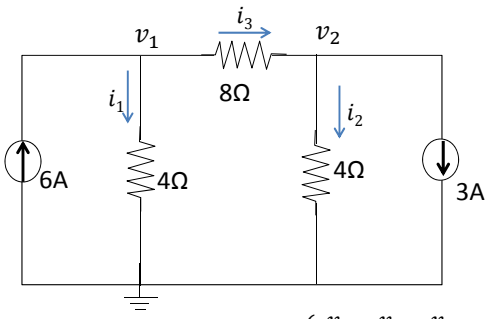
Practice problem 1:
Find v_1 and v_2 by using nodal analysis method



Solution:
Express resistor currents:

$$i_1 = \frac{v_1}{4}$$

$$i_2 = \frac{v_2}{4}$$

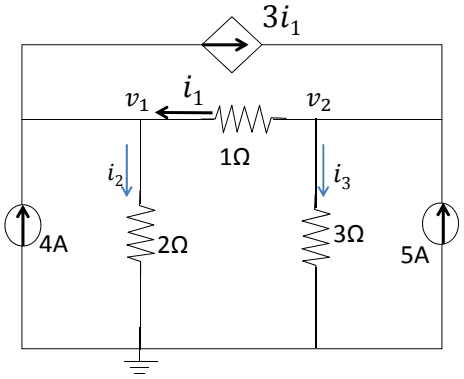
$$i_3 = \frac{v_1 - v_2}{8}$$

$$KCL\ at\ v_1:\ \ i_1 + i_3 = 6$$
$$at\ v_2:\ -i_2 + i_3 = 3$$
$$\Rightarrow \begin{cases} \frac{v_1}{4} + \frac{v_1 - v_2}{8} = 6 \\ -\frac{v_2}{4} + \frac{v_1 - v_2}{8} = 3 \end{cases}$$

$$\Rightarrow \begin{cases} 3v_1 - v_2 = 48 \\ v_1 - 3v_2 = 24 \end{cases} \Rightarrow \boxed{\begin{cases} v_1 = 15V \\ v_2 = -3V \end{cases}}$$

Practice problem 2:
Find i_1 by using nodal analysis method.

How many node voltages?



Solution:

$$i_1 = \frac{v_2 - v_1}{1}; \quad i_2 = \frac{v_1}{2}$$

$$i_3 = \frac{v_2}{3}$$

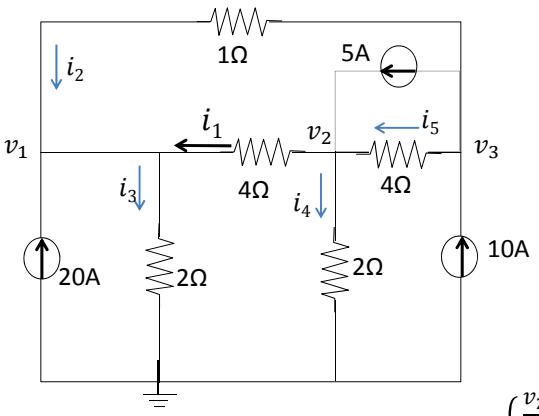
KCL at v_1: $i_1 - i_2 - 3i_1 = -4$
at v_2: $3i_1 - i_1 - i_3 = -5$

$$\Rightarrow \begin{matrix} 2i_1 + i_2 = 4 \\ 2i_1 - i_3 = -5 \end{matrix} \Rightarrow \begin{cases} 2\frac{v_2 - v_1}{1} + \frac{v_1}{2} = 4 \\ \frac{2(v_2 - v_1)}{1} - \frac{v_2}{3} = -5 \end{cases} \Rightarrow \begin{cases} -3v_1 + 4v_2 = 8 \\ -6v_1 + 5v_2 = -15 \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} v_1 = \frac{100}{9}V \\ v_2 = \frac{31}{3}V \end{cases}} \Rightarrow \boxed{i_1 = v_2 - v_1 = -\frac{7}{9}A = -0.778A}$$

R8

Practice problem 3: Form 3 equations for v_1, v_2, v_3



Solution:

$$i_1 = \frac{v_2 - v_1}{4}; \quad i_2 = \frac{v_3 - v_1}{1}$$

$$i_3 = \frac{v_1}{2}; \quad i_4 = \frac{v_2}{2};$$

$$i_5 = \frac{v_3 - v_2}{4}$$

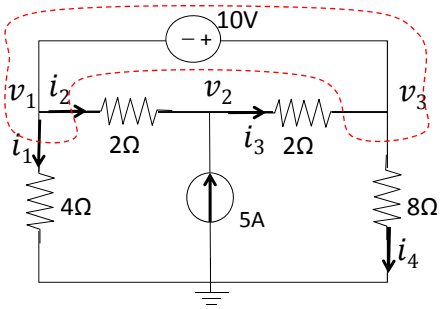
$KCL\ at\ v_1$: $i_1 + i_2 - i_3 = -20;$
$at\ v_2$: $-i_1 - i_4 + i_5 = -5;$
$at\ v_3$: $i_2 + i_5 = 5$

$$\begin{cases} \frac{v_2 - v_1}{4} + \frac{v_3 - v_1}{1} - \frac{v_1}{2} = -20 \\ -\frac{v_2 - v_1}{4} - \frac{v_2}{2} + \frac{v_3 - v_2}{4} = -5 \\ \frac{v_3 - v_1}{1} + \frac{v_3 - v_2}{4} = 5 \end{cases}$$

$$\begin{cases} -7v_1 + v_2 + 4v_3 = -80 \\ v_1 - 4v_2 + v_3 = -20 \\ -4v_1 - v_2 + 5v_3 = 20 \end{cases}$$

R9

Practice 4: Form 3 equations for the three node voltages



Solution:

$$i_1 = \frac{v_1}{4} \qquad i_2 = \frac{v_1 - v_2}{2}$$

$$i_3 = \frac{v_2 - v_3}{2} \qquad i_4 = \frac{v_3}{8}$$

Due to the floating voltage source, a supernode needs to be formed:

KCL at supernode: $-i_1 - i_2 + i_3 - i_4 = 0$
KCL at v_2: $i_2 - i_3 = -5;$
By the 10V source: $v_3 - v_1 = 10$

$$\begin{cases} -\frac{v_1}{4} - \frac{v_1 - v_2}{2} + \frac{v_2 - v_3}{2} - \frac{v_3}{8} = 0 \\ \frac{v_1 - v_2}{2} - \frac{v_2 - v_3}{2} = -5 \\ v_3 - v_1 = 10 \end{cases}$$

$$\begin{cases} -6v_1 + 8v_2 - 5v_3 = 0 \\ v_1 - 2v_2 + v_3 = -10 \\ -v_1 + v_3 = 10 \end{cases}$$

R9

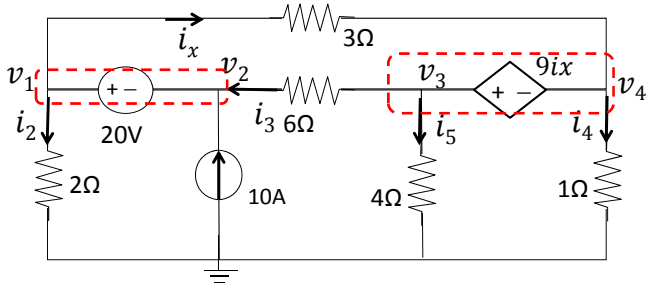
Practice 5: Form 4 equations for the four node voltages

Solution:

$i_x = \frac{v_1 - v_4}{3}$

$i_2 = \frac{v_1}{2}$ $\quad i_4 = \frac{v_4}{1}$

$i_3 = \frac{v_3 - v_2}{6}$ $\quad i_5 = \frac{v_3}{4}$



Due to the two floating voltage sources, two supernodes need to be formed:

KCL at supernode v_1v_2: $-i_2 - i_x + i_3 = -10$;

at supernode v_3v_4: $i_x - i_3 - i_4 - i_5 = 0$;

By 20V source: $v_1 - v_2 = 20$

By $9i_x$ source: $v_3 - v_4 = 9i_x$

$$\begin{cases} -\frac{v_1}{2} - \frac{v_1 - v_4}{3} + \frac{v_3 - v_2}{6} = -10 \\ \frac{v_1 - v_4}{3} - \frac{v_3 - v_2}{6} - \frac{v_4}{1} - \frac{v_3}{4} = 0 \\ v_1 - v_2 = 20 \\ v_3 - v_4 = 9\frac{v_1 - v_4}{3} \end{cases} \Rightarrow \begin{cases} -5v_1 - v_2 + v_3 + 2v_4 = -60 \\ 4v_1 + 2v_2 - 5v_3 - 16v_4 = 0 \\ v_1 - v_2 = 20 \\ 3v_1 - v_3 - 2v_4 = 0 \end{cases}$$

R9

Practice 6: Find v_1, v_2, v_3 *and* i_{dep}

Solution: $i_1 = \frac{v_1 - v_2}{3}$;

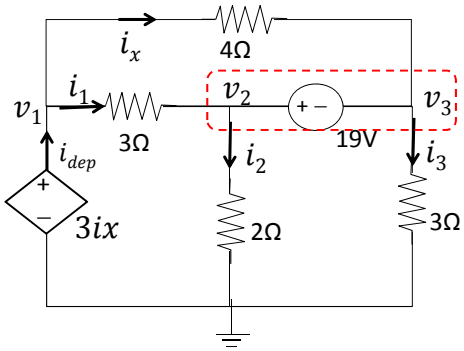
$i_2 = \frac{v_2}{2}$; $\quad i_3 = \frac{v_3}{3}$; $\quad i_x = \frac{v_1 - v_3}{4}$

Due to the floating voltage source, a supernode needs to be formed

KCL at supernode: $i_1 - i_2 - i_3 + i_x = 0$

By 19V source: $v_2 - v_3 = 19$

By $3i_x$ dep source: $v_1 = 3i_x$



$$\begin{cases} \frac{v_1 - v_2}{3} - \frac{v_2}{2} - \frac{v_3}{3} + \frac{v_1 - v_3}{4} = 0 \\ v_2 - v_3 = 19 \\ v_1 = 3\frac{v_1 - v_3}{4} \end{cases} \Rightarrow \begin{cases} 7v_1 - 10v_2 - 7v_3 = 0 \\ v_2 - v_3 = 19 \\ v_1 + 3v_3 = 0 \end{cases} \Rightarrow \begin{cases} v_1 = 15V \\ v_2 = 14V \\ v_3 = -5V \end{cases}$$

$$i_{dep} = i_1 + i_x = \frac{15 - 14}{3} + \frac{15 + 5}{4}A = \frac{16}{3}A = 5.333A$$