Practice 1: Solve $\ddot{v} + 5\dot{v} + 6v = 18, \quad v(0) = 0; \quad \dot{v}(0) = 4$

Solution: $\alpha = \frac{5}{2} = 2.5; \;\; \omega_0 = \sqrt{6}\,, \quad \alpha > \omega_0, \;\; Case\ 1.$

$$s_1, s_2 = -2.5 \pm \sqrt{2.5^2 - 6} = -2.5 \pm 0.5 = -2, -3$$

$$v(\infty) = \frac{18}{6} = 3.$$

The general solution: $v(t) = 3 + A_1 \mathrm{e}^{-2\mathrm{t}} + \mathrm{A}_2 \mathrm{e}^{-3\mathrm{t}}$

To satisfy initial condition:

$\mathrm{v}(0) = 3 + \mathrm{A}_1 + \mathrm{A}_2 = 0;$

$\dot{v}(0) = -2A_1 - 3A_2 = 4\,;$

$\rightarrow \;\; A_1 = -5, A_2 = 2;$

Final solution: $v(t) = 3 - 5e^{-2t} + 2e^{-3t}$

Practice 2: Solve $\ddot{v} + 4\dot{v} + 4v = 8, \quad v(0) = 1; \quad \dot{v}(0) = -1$

Solution: $\alpha = \frac{4}{2} = 2; \;\; \omega_0 = 2 \quad \alpha = \omega_0, \;\; Case\ 2$

$$v(\infty) = \frac{8}{4} = 2.$$

The general solution: $v(t) = 2 + (\mathrm{A}_1 + \mathrm{A}_2 \mathrm{t})\mathrm{e}^{-2\mathrm{t}}$

To satisfy initial condition:

$\mathrm{v}(0) = 2 + \mathrm{A}_1 = 1;$

$\dot{v}(0) = -2A_1 + A_2 = -1\,;$

$\rightarrow \;\; A_1 = -1\; A_2 = -3;$

Final solution: $v(t) = 2 - (1 + 3t)e^{-2t}$

Practice 3: Solve $\ddot{v}+4\dot{v}+13v=39, \quad v(0)=1; \quad \dot{v}(0)=-1$

Solution: $\alpha=\frac{4}{2}=2; \ \ \omega_0=\sqrt{13}, \quad \alpha<\omega_0, \ \ Case\ 3.$

$$\omega_d=\sqrt{13-4}=3$$

$$v(\infty)=\frac{39}{13}=3.$$

The general solution:

$$v(t)=3+\mathrm{e}^{-2\mathrm{t}}(\mathrm{B}_1\cos 3\mathrm{t}+\mathrm{B}_2\sin 3\mathrm{t})$$

To satisfy initial condition:

$\mathrm{v}(0)=3+\mathrm{B}_1=1;$

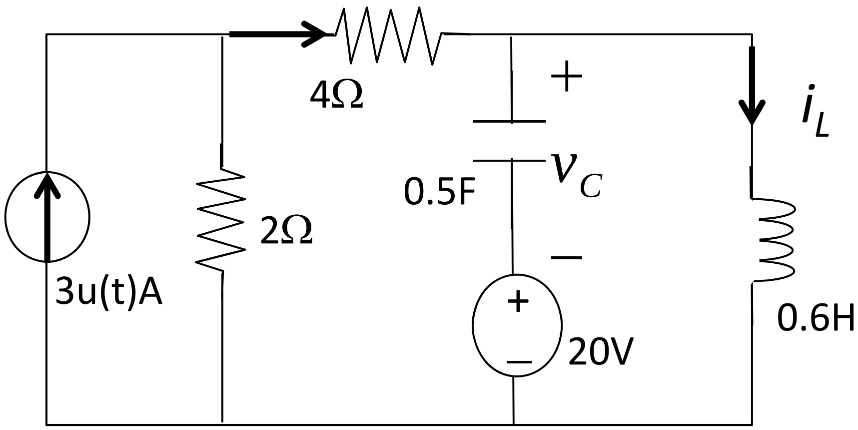
$\dot{v}(0)=-2B_1+3B_2=-1\ ;$

$\rightarrow \ \ B_1=-2, B_2=-\frac{5}{3};$

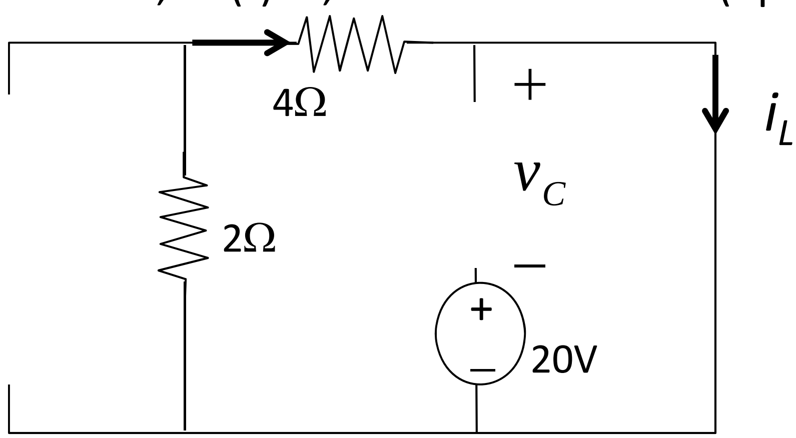
or $B_2=\frac{V_1+\alpha\ B_1}{\omega_d}=\frac{-1-4}{3}=-\frac{5}{3}$

Final solution: $v(t)=3-e^{-2t}(2cos3t\ e^{-3t}+\frac{5}{3}sin3t\ )$

Practice 4: Find $v_C(0^+), i_L(0^+), \dfrac{dv_C(0^+)}{dt}, \dfrac{di_L(0^+)}{dt}$

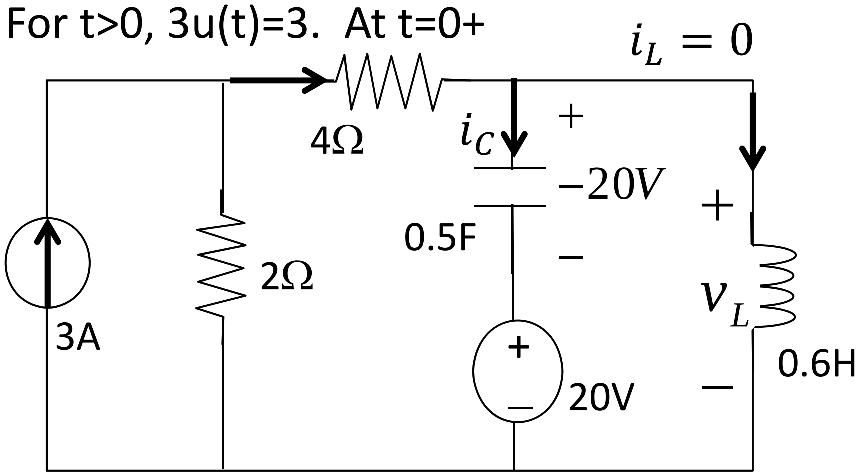


Fot t<0, 3u(t)=0, current source off (open), C open, L short



$i_L(0) = 0;$
$v_c(0) = -20V$ by KVL
No voltage in resistors
Since 20V does not
Supply power

For t>0, 3u(t)=3. At t=0+



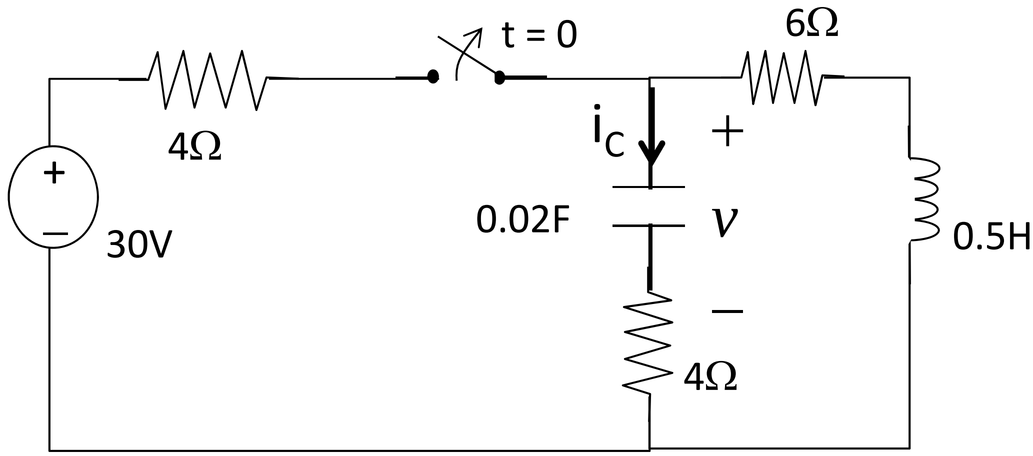
By KVL,
$v_L(0^+) = -20 + 20 = 0V$
4Ω and 2Ω in parallel
By current division

$$i_{4\Omega} = \frac{2}{4+2}3 = 1A;$$

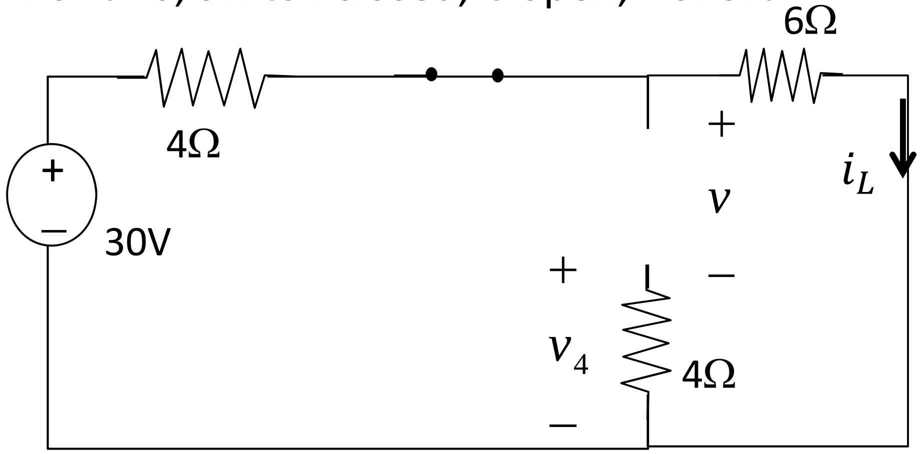
By KCL,
$i_C(0^+) = i_{4\Omega} - i_L = 1A$

Thus, $\dfrac{dv_c(0^+)}{dt} = \frac{1}{C}i_c(0^+) = \frac{1}{0.5} = 2V/s$, $\dfrac{di_L(0^+)}{dt} = \frac{1}{L}v_L(0^+) = 0$

Practice 5: Find v(t) for t > 0.



For t< 0, switch closed, C open, L short

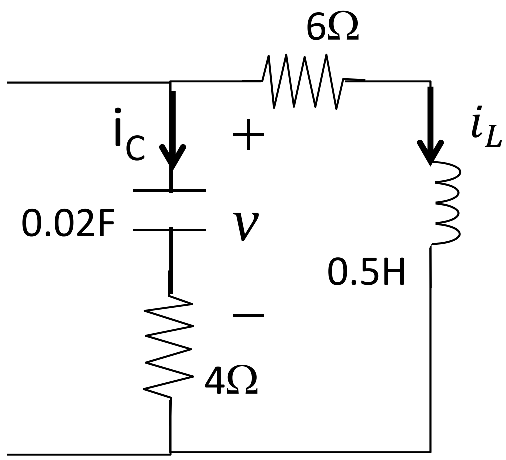


since $v_4 = 0$,
v is the voltage across 6Ω.
By voltage division,

$$v(0) = \frac{6}{6+4}30 = 18V$$

$$i_L(0) = \frac{30}{10} = 3A$$

For t > 0.



$$Since\ i_L = -i_c, \quad i_c(0+) = -3A$$

$$\frac{dv(0^+)}{dt} = -\frac{3}{0.02} = -150V/s$$

$$\alpha = \frac{4+6}{2\times 0.5} = 10, \omega_0 = \sqrt{\frac{1}{0.01}} = 10$$

Case 2. Since v(∞) = 0,

$$v(t) = (A_1 + A_2 t)e^{-10t}$$

To satisfy initial condition

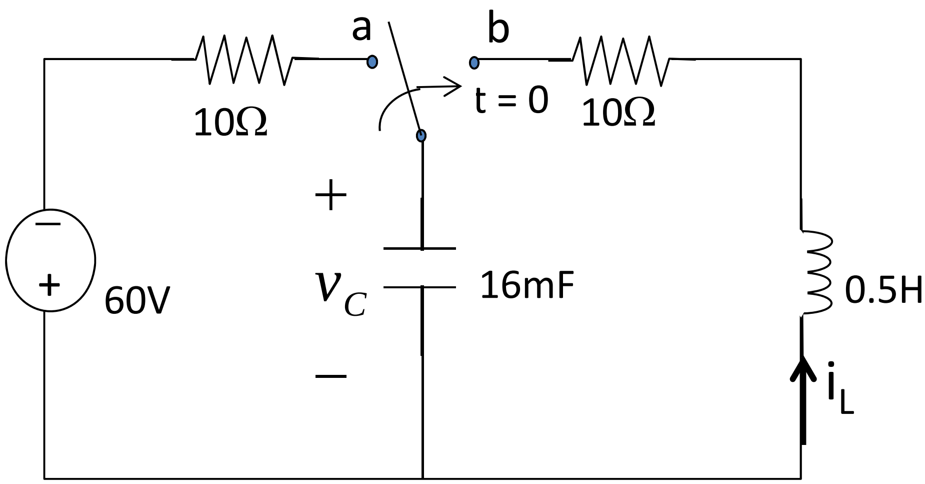
$$v(0) = A_1 = 18;$$

$$\dot{v}(0) = -10A_1 + A_2 = -150$$

$$\rightarrow \quad A_2 = 30$$

Finally, $v(t) = (18 + 30t)e^{-10t}V$

Practice 6: The switch is at position a for a long time before swinging to position b at t = 0. Find $i_L(t)$, $v_C(t)$ for t > 0.



$$v_c(0) = -60V, \qquad i_L(0) = 0A, \qquad \frac{dv_c(0^+)}{dt} = \frac{0V}{s}$$

$$\alpha = 10, \omega_0 = \sqrt{125}, \qquad \alpha < \omega_0$$

Case 3: $\omega_d = \sqrt{125 - 100} = 5$

Since $v_c(\infty) = 0, v(t) = e^{-10t}(B_1 \cos 5t + B_2 \sin 5t)$

To satisfy initial conditions:

$B_1 = -60$

$-10B_1 + 5B_2 = 0, B_2 = -120$

Or, use $B_2 = \frac{\dot{v}(0)+\alpha B_1}{\omega_d} = \frac{10\times(-60)}{5}$=-120

Thus $v(t) = e^{-10t}(-60cos5t - 120sin5t)$ V

$$i_L(t) = i_{c(t)} = C\frac{dv_c}{dt}$$

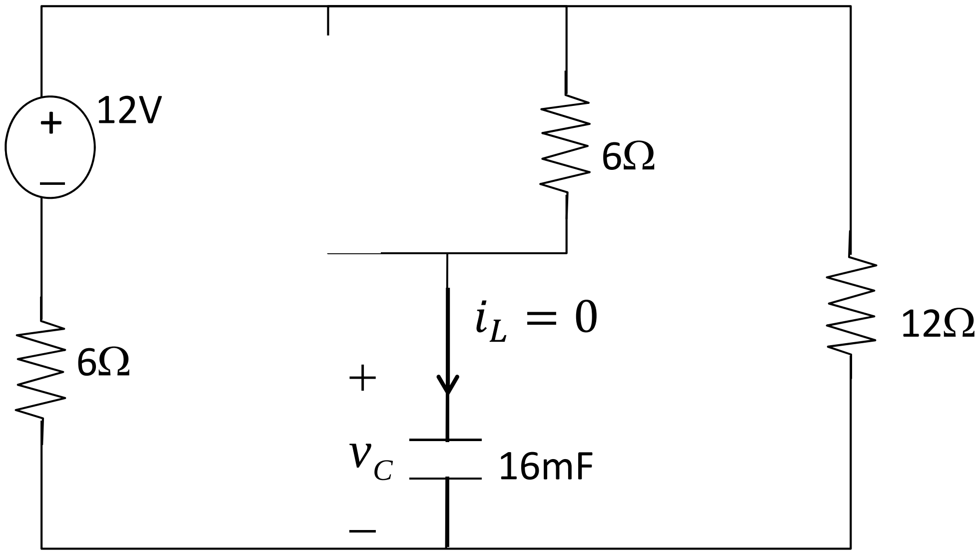
$$= 0.016((-10)e^{-10t}(-60cos5t - 120sin5t)$$

$$+e^{-10t}(300sin5t - 600cos5t))$$

$$=24e^{-10t}sin5t\ A$$

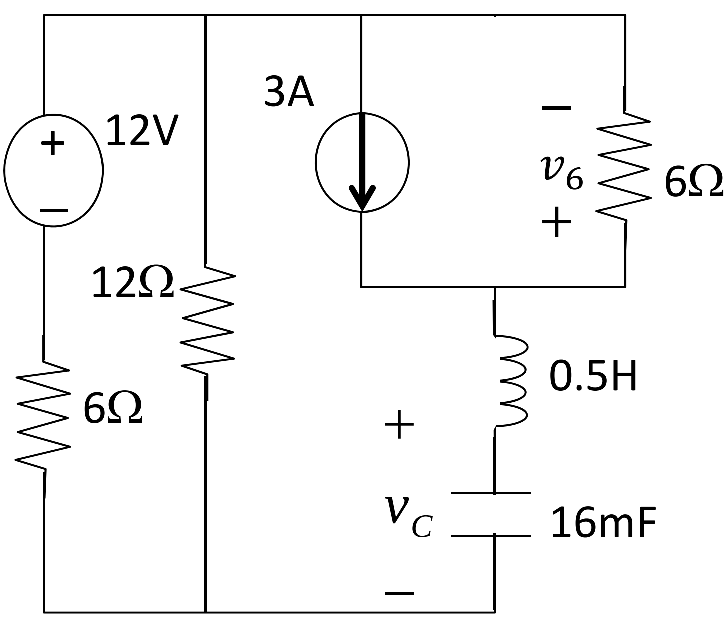
Practice 7: Find $v_C(t)$ for t > 0.

For t < 0, 3u(t)A=0, current source open, C open , L short



$$v_c(0) = v_R = \frac{12}{6+12} \times 12 = 8V, \qquad i_L(0) = 0, \qquad \frac{dv_c(0)}{dt} = 0$$

Fot t>0, 3u(t)=3. To see connection better, move 12 Ω to the left



$R_{th} = 6 + 12//6 = 10\Omega$

Under DC condition,

$v_R = 8V, v_6 = 18V,$

$v_c(\infty) = V_{th} = 8 + 18 = 26V$

$\alpha = \dfrac{10}{2 \times 0.5} = 10;$

$\omega_0 = 1/(0.5 \times 0.016)^{0.5}$

$= \sqrt{125}\,,$

$\alpha < \omega_0, \qquad case\ 3$

$\omega_d = \sqrt{125 - 100} = 5$

$\mathrm{v_c(t)} = 26 + \mathrm{e^{-10t}}(\mathrm{B_1 cos\ 5t + B_2 sin\ 5t})$

To satisfy initial condition:

$v_c(0) = 26 + B_1 = 8$

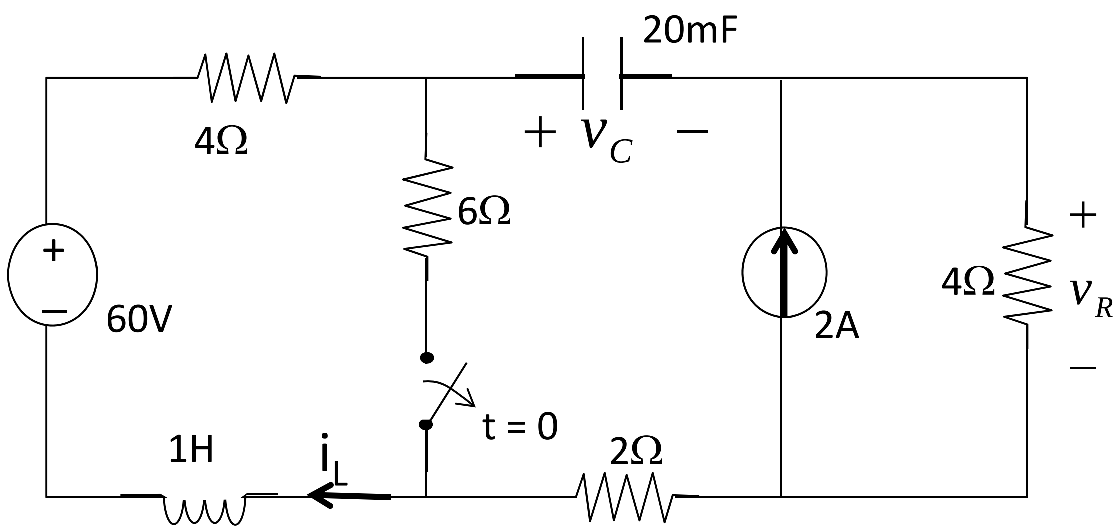
$\dot{v}_c(0) = -10B_1 + 5B_2 = 0$

Solving equations to get $B_1 = -18; B_2 = -36$

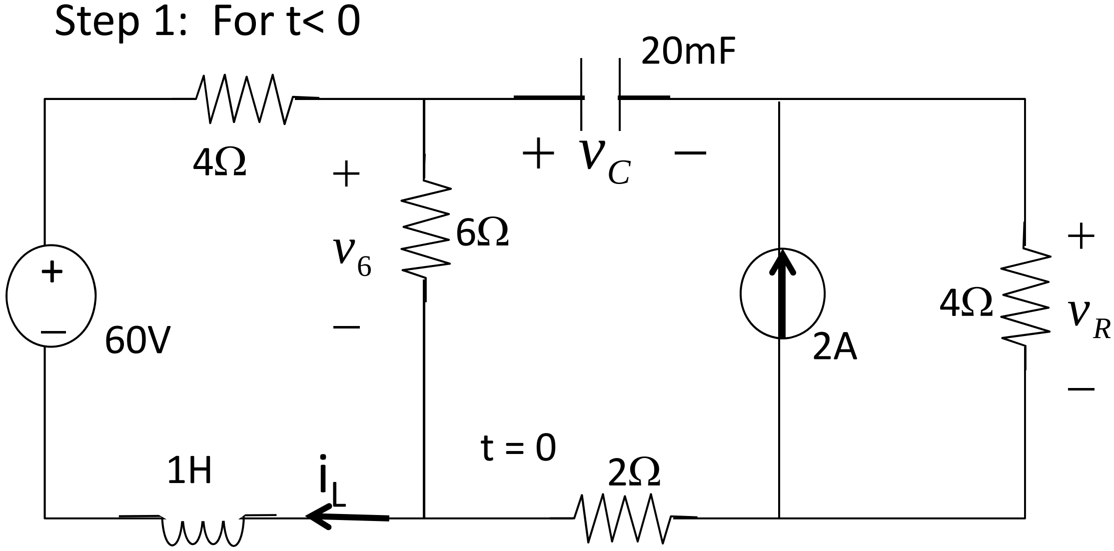
Step response:

$$\mathrm{v_c(t)} = 26 + \mathrm{e}^{-10t}(-18\cos 5t\ - 36\sin 5t)V$$

Practice 8: Find $i_L(t)$, $v_C(t)$, $v_R(t)$ for t > 0.



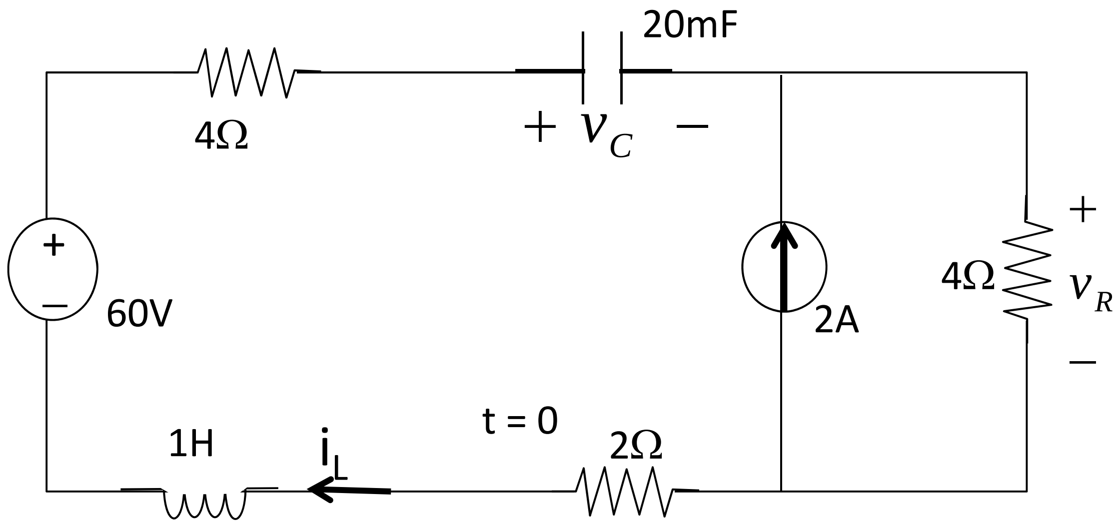
Step 1: For t< 0



Since C open, L short, $i_L(0) = \frac{60}{6+4} = 6A,\ v_6 = 36V,\ v_R = 8V$

No current and voltage drop in 2Ω , $v_c(0) = v_6 - v_R = 28V$

For t > 0,



Step 2: $i_c(0+) = i_L(0) = 6A,\ \frac{dv_c(0^+)}{dt} = \frac{6}{0.02} = 300V/s$

$R_{th} = 4 + 2 + 4 = 10\Omega, \qquad V_{th} = v_c(\infty) = 60 - 8 = 52V$

Step 3: Math computation

$\alpha = \frac{10}{2} = 5, \qquad \omega_0 = \frac{1}{(0.02)^{0.5}} = \sqrt{50}$

$\alpha < \omega_0 , \qquad Case\ 3, \qquad \omega_d = 5:$

General solution: $v_c(t) = 52 + e^{-5t}(B_1 \cos 5t + B_1 \sin 5t\ )$

By initial condition:

$v_c(0) = 52 + B_1 = 28;$

$\dot{v}_c(0) = -5B_1 + 5B_2 = 300;$

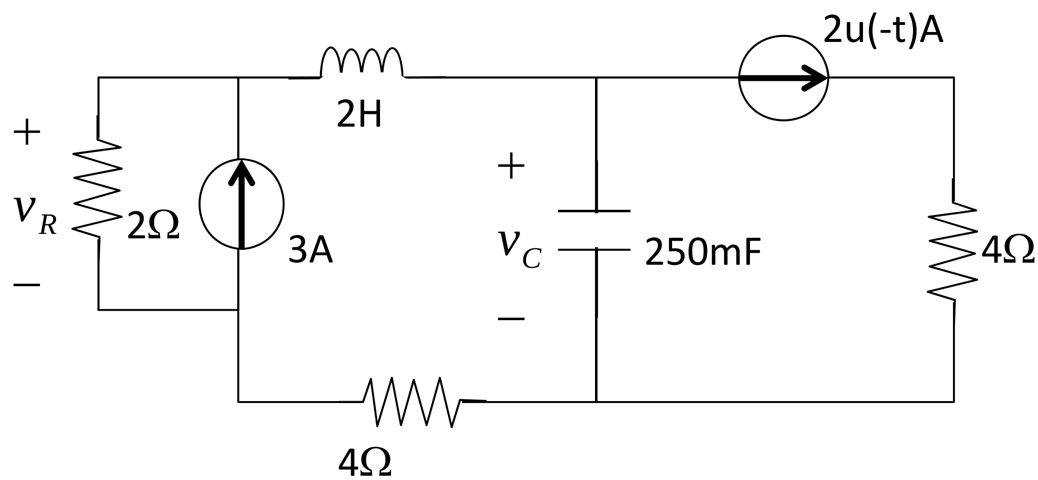
$B_1 = -24, \qquad B_2 = 36;$

$v_c(t) = 52 + e^{-5t}(-24 \cos 5t + 36 \sin 5t\ )V$

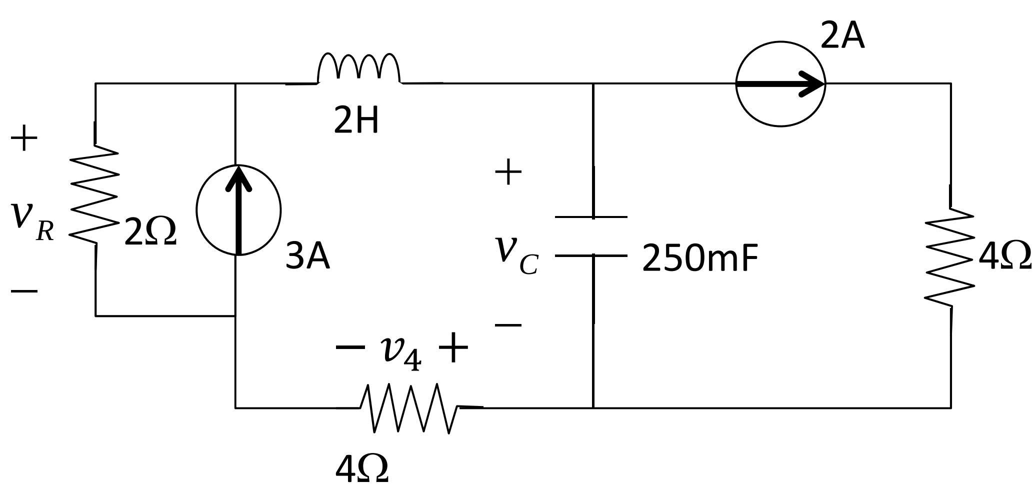
Other variables: $i_L(t) = i_c(t) = C\frac{dv_c}{dt}$

$$v_R(t) = 4\left(2 + C\frac{dv_c}{dt}\right) = 8 + e^{-5t}(24 \cos 5t\ - 4.8 \sin 5t\ )V$$

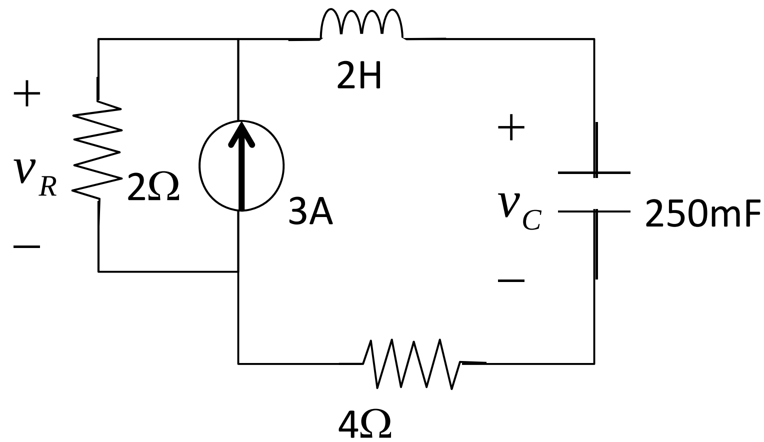
Practice 9: Find $v_C(t)$ and $v_R(t)$ for t > 0.



Step 1: t<0. 2u(-t)=2



$$v_R = 2(3-2) = 2V;\ \ v_4 = 2(4) = 8V;$$
$$v_c(0) = v_R - v_4 = -6V,\ \ i_L(0) = 2A$$



Step 2: t>0. 2u(-t)=0,

$i_c(0+) = i_L(0) = 2A;\ \dot{v}_c(0+) = \frac{2}{0.25} = 8V/s$

$$v_{th} = v_c(\infty) = 6V; \quad R_{th} = 6\Omega$$

Step 3: Math computation

$\alpha = \frac{6}{(2\times 2)} = 1.5;\ \omega_0 = \sqrt{2}$

$$\alpha > \omega_0, \qquad Case\ 1$$

$$s_1, s_2 = -1.5 \pm \sqrt{1.5^2 - 2} = -1, -2$$

General solution:

$$v_c(t) = 6 + A_1 e^{-t} + A_2 e^{-2t}$$

$$v_c(0) = 6 + A_1 + A_2 = -6$$

$$\dot{v}(0+) = -\mathrm{A}_1 - 2\mathrm{A}_2 = 8;$$

$$A_1 = -16, \qquad A_2 = 4$$

$$v_c(t) = 6 - 16e^{-t} + 4e^{-2t}\ V$$

For $v_R(t)$, $\quad v_R(t) = 2(3 - i_c) = 2\left(3 - C\frac{dv}{dt}\right)$

$$= 6 - 8e^{-t} + 4e^{-2t}V$$