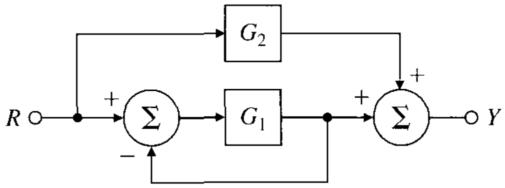
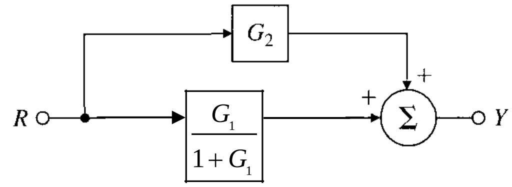
3.19 Find the transfer functions for the block diagrams in Fig. 3.54:



(a)

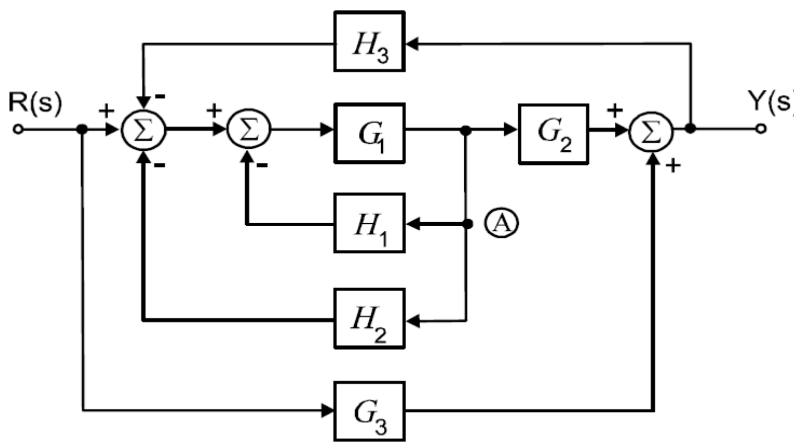


(a)

Solution: Simplify the block diagram as above,

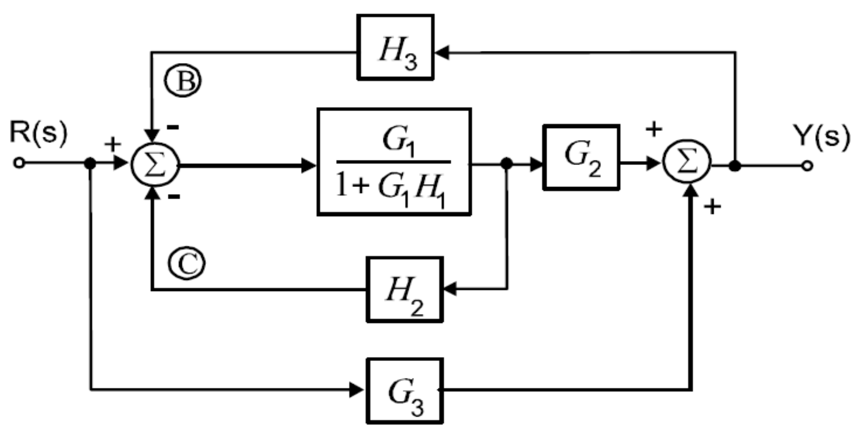
$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_1}{1+G_1} + G_2$$

3.22 Use block-diagram algebra to determine the transfer function between R(s) and Y(s) in Fig. 3.57.



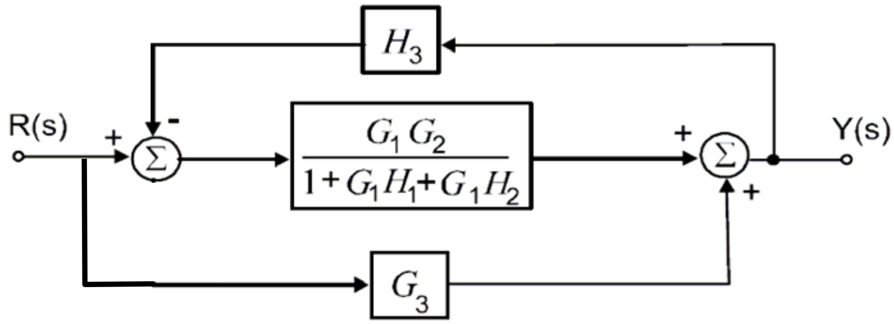
Block diagram for Fig. 3.57

Solution: Move node A and close the loop:



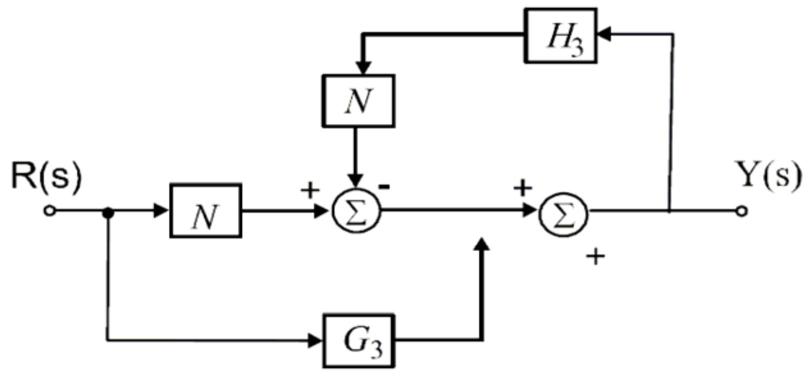
Block diagram for Fig. 3.57: reduced

Add signal B, close loop and multiply before signal C.



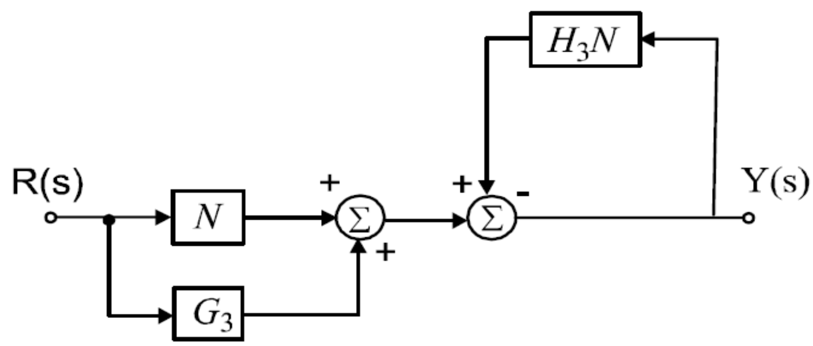
Block diagram for Fig. 3.57: reduced

Move middle block N past summer.



Block diagram for Fig. 3.57: reduced

Now reverse order of summers and close each block separately.



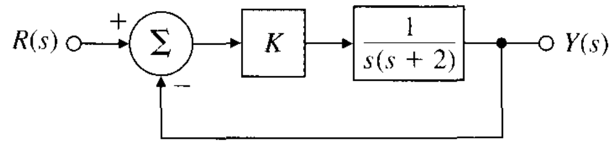
Block diagram for Fig. 3.57: reduced

$$\frac{Y}{R} = \overbrace{(N + G_3)}^{feedforward}\underbrace{(\frac{1}{1 + NH_3})}_{feedback}$$

$$\frac{Y}{R} = \frac{G_1G_2 + G_3(1 + G_1H_1 + G_1H_2)}{1 + G_1H_1 + G_1H_2 + G_1G_2H_3}$$

3.24 For the unity feedback system shown in Fig. 3.59, specify the gain K of the proportional controller so that the output $y(t)$ has an overshoot of no more than 10% in response to a unit step.

Figure 3.59
Unity feedback system for Problem 3.24



Solution:

$$\frac{Y(s)}{R(s)} = \frac{K}{s^2 + 2s + K} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{K},$$

$$\zeta = \frac{2}{2\omega_n} = \frac{1}{\sqrt{K}} \qquad (1)$$

In order to have an overshoot of no more than 10%,

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \le 0.10$$

Solving for ζ,

$$\zeta = \sqrt{\frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}} \ge 0.591$$

Using (1) and the solution for ζ,

$$K = \frac{1}{\zeta^2} \le 2.86$$

$$\therefore \ 0 \le K \le 2.86$$

3.28 The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s\ +\ 2)}$$

The desired system response to a step input is specified as peak time $t_p = 1$ sec and overshoot $M_p = 5\%$.

(a) Determine whether both specifications can be met simultaneously by selecting the right value of K.

(b) Sketch the associated region in the s-plane where both specifications are met, and indicate what root locations are possible for some likely values of K.

(c) Pick a suitable value for K, and use MATLAB to verify that the specifications are satisfied.

Solution:

(*a*)

$$T(s)=\frac{Y(s)}{R(s)}=\frac{G(s)}{1+G(s)}=\frac{K}{s^2+2s+K}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Equate the coefficients:

$$2=2\zeta\omega_n, \qquad K=\omega_n^2 \qquad (*)$$

$$\Rightarrow\ \omega_n=\sqrt{K} \qquad \zeta=\frac{1}{\sqrt{K}}$$

We would need:

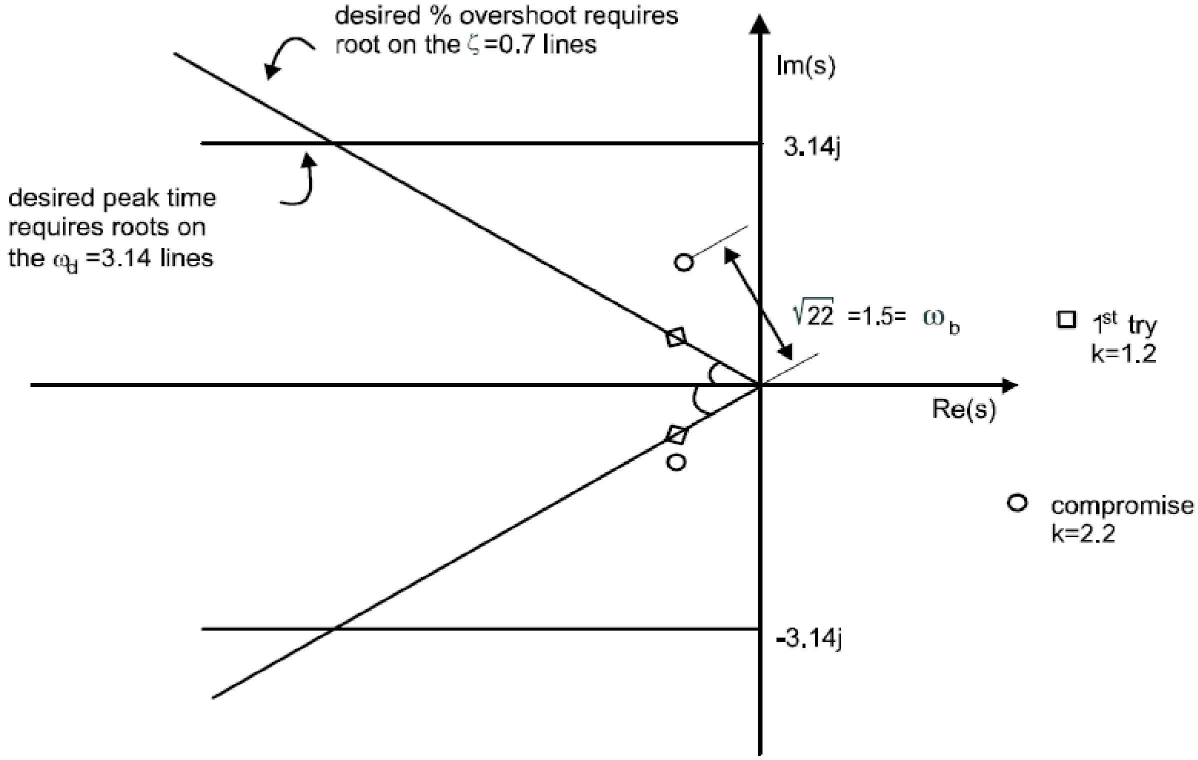
$$\frac{M_p\%}{100}=0.05=e^{-\pi\zeta/\sqrt{1-\zeta^2}} \qquad \Rightarrow\ \zeta=0.69$$

$$t_p=1\ \sec=\frac{\pi}{\omega_d}=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad \Rightarrow\ \omega_n=4.34$$

But the combination ($\zeta=0.69,\ \omega_n=4.34$) that we need is not possible by varying K alone.

Observe that from equations (*) $\zeta\omega_n=1\neq 0.69\times 4.34$

(*b*)



s-plane regions

(c) Now we wish to have:

$$M_p^* = r \times 0.05 = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \qquad t_p^* = r \times 1 \text{ sec} = \frac{\pi}{\omega_d} \qquad (**)$$

where $r \equiv$ relaxation factor.

Recall the conditions of our system:

$$\omega_n = \sqrt{K} \qquad \zeta = \frac{1}{\sqrt{K}}$$

replace ω_n and ζ in the system (**):

$$\Rightarrow \quad e^{-\pi/\sqrt{K-1}} = r \times 0.05, \qquad \frac{\pi}{\sqrt{K-1}} = r$$

$$\Rightarrow \quad r \times 0.05 = e^{-r}, \qquad \Rightarrow \quad r \cong 2.21$$

$$K = 1 + \frac{\pi^2}{r^2} \qquad \Rightarrow \quad K = 3.02$$

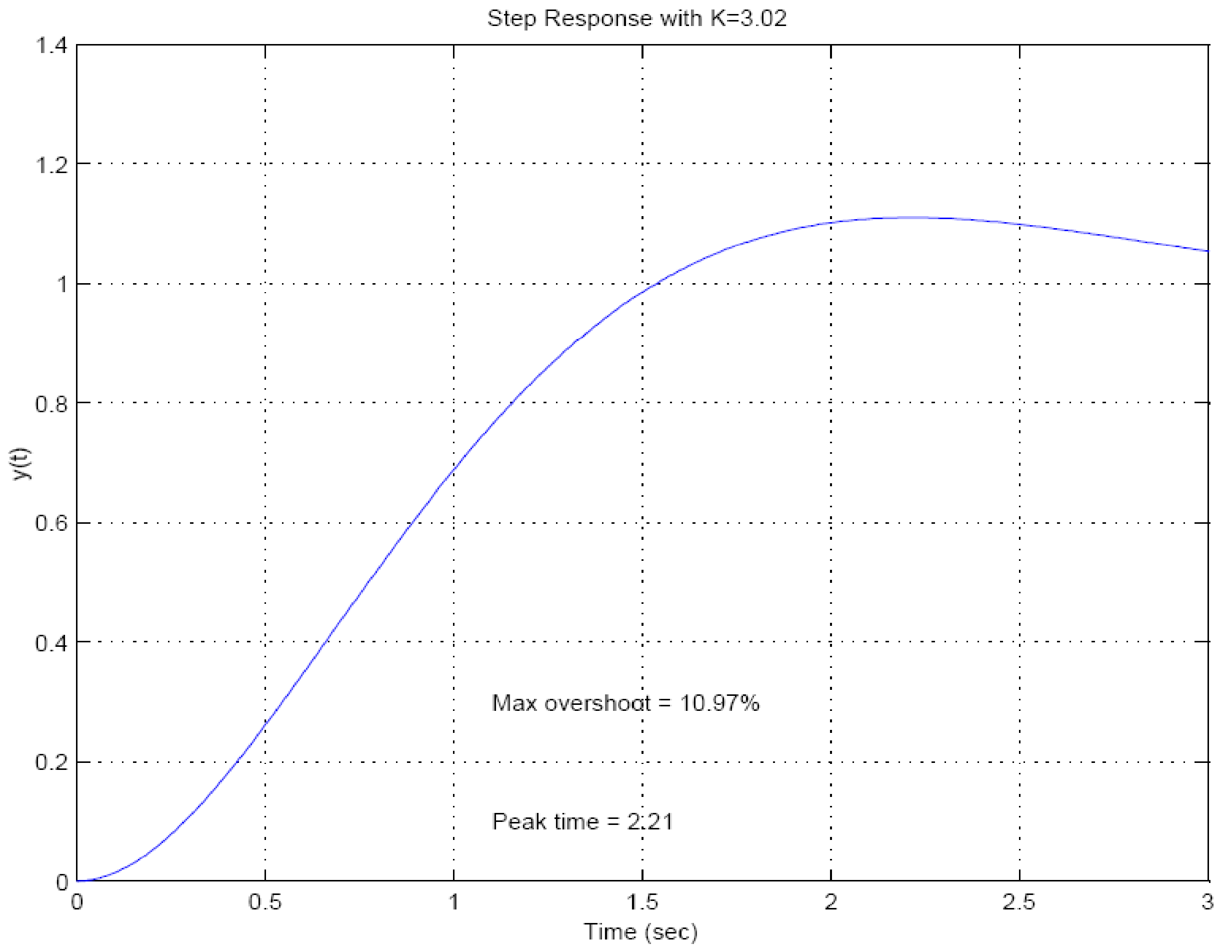
then with $K = 3.02$ we will have:

$$M_p^* = rM_p = 2.21 \times 0.05 = 0.01$$

$$t_p^* = rt_p = 2.21 \times 1 \text{ sec} = 2.21 \text{ sec}$$

Note: * denotes actual location of closed-loop roots.

```
% Problem 3.28
K=3.02;
num=[K];
den=[1, 2, K];
sys=tf(num,den);
t=0:.01:3;
y=step(sys,t);
plot(t,y);
yss = dcgain(sys);
Mp = (max(y) - yss)*100;
% Finding maximum overshoot
msg_overshoot = sprintf('Max overshoot = %3.2f%%', Mp);
% Finding peak time
idx = max(find(y==(max(y))));
tp = t(idx);
msg_peaktime = sprintf('Peak time = %3.2f', tp);
xlabel('Time (sec)');
ylabel('y(t)');
msg_title = sprintf('Step Response with K=%3.2f',K);
title(msg_title);
text(1.1, 0.3, msg_overshoot);
text(1.1, 0.1, msg_peaktime);
grid on;
```



Problem 3.28: Closed-loop step response

A: Use matlab to plot the step response for ξ=0.2,0.4,0.6,0.8 for system $H(s) = \frac{1}{s^2 + 2\xi s + 1}$. Choose the final time as 15 seconds.

Solution:

```
% Problem 3.A
linespec=['r','g','b','k'];
t=0:.01:15;
for i=1:4
   ci=i*0.2;
   num=[1];
   den=[1, 2*ci, 1];
   sys=tf(num,den);
   y=step(sys,t);
   plot(t,y,linespec(i));
   hold on;
```

```
end
legend('damping ratio = 0.2','damping ratio = 0.4','damping ratio = 0.6','damping ratio = 0.8');
xlabel('Time (sec)');
ylabel('y(t)');
title('Step Response with damping ratio = 0.2, 0.4, 0.6, 0.8');
grid on;
```