4.4 The DC-motor speed control in Fig. 4.38 is described by the differential equation

$$\dot{y} + 60y = 600v_a - 1500w,$$

where y is the motor speed, v_a is the armature voltage, and w is the load torque. Assume the armature voltage is computed using the PI control law

$$v_a = \left(k_p e + k_I \int_0^t e dt\right),$$

where $e = r - y$.

(a) Compute the transfer function from W to Y as a function of k_p and k_I.

(b) Compute values for k_p and k_I so that the characteristic equation of the closed-loop system will have roots at $-60 \pm 60j$.

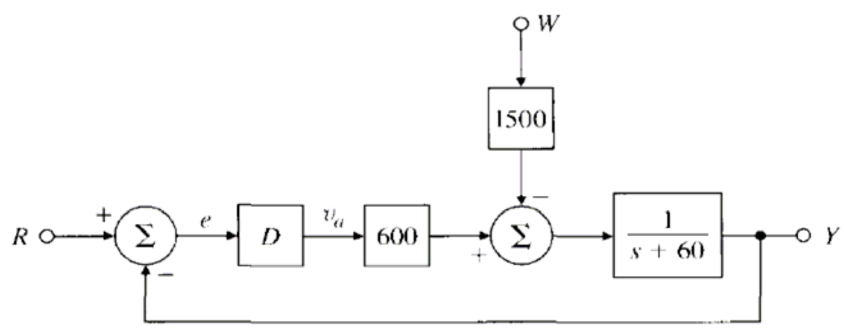

Figure 4.38: Unity feedback system with preÞlter for Problem 4.4

Solution:

(a) Transfer function: Set $R(s) = 0,$ then $E(s) = -Y(s)$

$$Y(s) = \frac{-600\left(k_p + \frac{k_I}{s}\right)Y(s) - 1500W(s)}{(s+60)}$$

$$\frac{Y(s)}{W(s)} = \frac{-1500s}{s^2 + 60(1+10k_p)s + 600k_I}$$

(b) For roots at $s_{1,2} = -60 \pm j60,$

$$\begin{cases} s_1 + s_2 = -60(1+10k_p) = -120 \\ s_1 s_2 = 7200 = 600k_I \end{cases} \Rightarrow \quad k_p = 0.1, \qquad k_I = 12$$

4.26 Consider the system shown in Fig. 4.49 with PI control.

(a) Determine the transfer function from R to Y.

(b) Determine the transfer function from W to Y.

(c) Use Routh's criteria to find the range of $(k_p,\ k_I)$ for which the system is stable.

(d*) Pick k_p and k_I so that the closed-loop system is stable. What is the steady state error when $r(t) = 1(t)$ and $w(t) = 1(t)$?

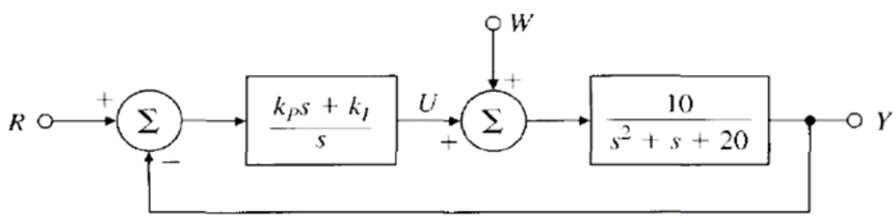

Figure 4.49: Control system for Problem 4.26

Solution:

$$(a)\ \frac{Y(s)}{R(s)} = \frac{\frac{10}{s^2+s+20}\frac{k_p s+k_I}{s}}{1+\frac{10}{s^2+s+20}\frac{k_p s+k_I}{s}} = \frac{10(k_p s+k_I)}{s^3+s^2+10(2+k_p)s+10k_I}$$

$$(b)\ \frac{Y(s)}{W(s)} = \frac{\frac{10}{s^2+s+20}}{1+\frac{10}{s^2+s+20}\frac{k_p s+k_I}{s}} = \frac{10s}{s^3+s^2+10(2+k_p)s+10k_I}$$

(c) The characteristic equation is $s^3+s^2+10(2+k_p)s+10k_I=0$. The Routh's array is

$$\begin{array}{lll} s^3: & 1 & 10(2+k_p) \\ s^2: & 1 & 10k_I \\ s^1: & 10(2+k_p-k_I) & \\ s^0: & 10k_I & \end{array}$$

For stability we must have $k_I > 0$ and $k_p > k_I - 2$.

(d^*) Choose $k_I = 1,\ \ k_p = 1,$ The transfer functions become,

$$\frac{Y(s)}{R(s)} = \frac{10s+10}{s^3+s^2+30s+10}, \qquad \frac{Y(s)}{W(s)} = \frac{10s}{s^3+s^2+30s+10}$$

Since $\frac{Y(0)}{R(0)} = 1,\ \ \frac{Y(s)}{W(s)} = 0,$ the steady state error is 0.

As a verification, let $r(t) = 1(t)$ and $w(t) = 1(t),$ using matlab, we can get the following response,