4.12 Consider the second-order plant

$$G(s) = \frac{1}{(s+1)(5s+1)}$$

($a$) Determine the system type and error constant with respect to tracking polynomial reference inputs of the system for P, PD, and PID controllers (as configured in Fig.4.28). Let $k_p = 19$, $k_I = 0.5$, and $k_D = \frac{4}{19}$.

($b$) Determine the system type and error constant of the system with respect to disturbance inputs for each of the three regulators in part ($a$) with respect to rejecting polynomial disturbances $w(t)$ at the input to the plant.

($c$) Is this system better at tracking references or rejecting disturbances? Explain your response briefly.

($d$) Verify your results for parts ($a$) and ($b$) using MATLAB by plotting unit step and ramp responses for both tracking and disturbance rejection.

**Solution**:

($a$) Given $k_p = 19,\ k_I = 0.5,\ k_D = \frac{4}{19}$,

- P control: $D(s) = k_p = 19,\ \ D(s)G(s) = \frac{19}{(s+1)(5s+1)}$, No poles at $s = 0$, Type 0

$$K_p = \lim_{s\to 0} D(s)G(s) = 19$$

- PD control: $D(s) = k_D s + k_p = \frac{4}{19}s + 19,\ \ D(s)G(s) = \frac{\frac{4}{19}s + 19}{(s+1)(5s+1)}$, No poles at $s = 0$, Type 0

$$K_p = \lim_{s\to 0} D(s)G(s) = 19$$

- PID control: $D(s) = k_D s + k_p + k_I / s = \frac{4}{19}s + 19 + 0.5/s,\ \ D(s)G(s) = \frac{\frac{4}{19}s^2 + 19s + 0.5}{s(s+1)(5s+1)}$,

$s = 0$ is 1-order pole, Type 1, $K_v = \lim_{s\to 0} sD(s)G(s) = 0.5$

($b$) $T_w(s) = \frac{E(s)}{W(s)} = \frac{-G(s)}{1 + D(s)G(s)}$

- P control: $D(s) = k_p = 19,\ \ T_w(s) = \frac{-1}{(s+1)(5s+1)+19}$, No zeros at $s = 0$, Type 0

$$K_{p,w} = \frac{1}{\lim_{s\to 0} T_w(s)} = -20$$

- PD control: $D(s) = k_D s + k_p = \frac{4}{19}s + 19,\ \ T_w(s) = \frac{-1}{(s+1)(5s+1) + \frac{4}{19}s + 19}$,

$$\text{No zeros at } s=0,\ \text{Type 0},\ K_{p,w}=\frac{1}{\lim_{s\to 0} T_w(s)}=-20$$

- PID control: $D(s)=k_D s+k_p+k_I/s=\frac{4}{19}s+19+0.5/s,$

$$T_w(s)=\frac{-s}{s(s+1)(5s+1)+\frac{4}{19}s^2+19s+0.5}, \qquad s=0 \text{ is 1-order zero, Type 1,}$$

$$K_{v,w}=\frac{1}{\lim_{s\to 0} T_w(s)/s}=-0.5$$

($c$) There is reduced oscillatory behavior brought on by addition of the derivative term, and increased oscillatory but lower error brought on by the integral term.

($d$) Reference Tracking:

$$\frac{E(s)}{R(s)}=\frac{1}{1+D(s)G(s)}=\begin{cases}\dfrac{5s^2+6s+1}{5s^2+6s+20}, & \text{P Control}\\[2ex] \dfrac{5s^2+6s+1}{5s^2+\frac{118}{19}s+20}, & \text{PD control}\\[2ex] \dfrac{5s^3+6s^2+s}{5s^3+\frac{118}{19}s^2+20s+0.5}, & \text{PID control}\end{cases}$$

Disturbance Rejection:

$$\frac{E(s)}{W(s)}=\frac{-G(s)}{1+D(s)G(s)}=\begin{cases}\dfrac{-1}{5s^2+6s+20}, & \text{P Control}\\[2ex] \dfrac{-1}{5s^2+\frac{118}{19}s+20}, & \text{PD control}\\[2ex] \dfrac{-s}{5s^3+\frac{118}{19}s^2+20s+0.5}, & \text{PID control}\end{cases}$$

For Ramp Response, Use $y=lsim(sys,t,t)$ in MATLAB.

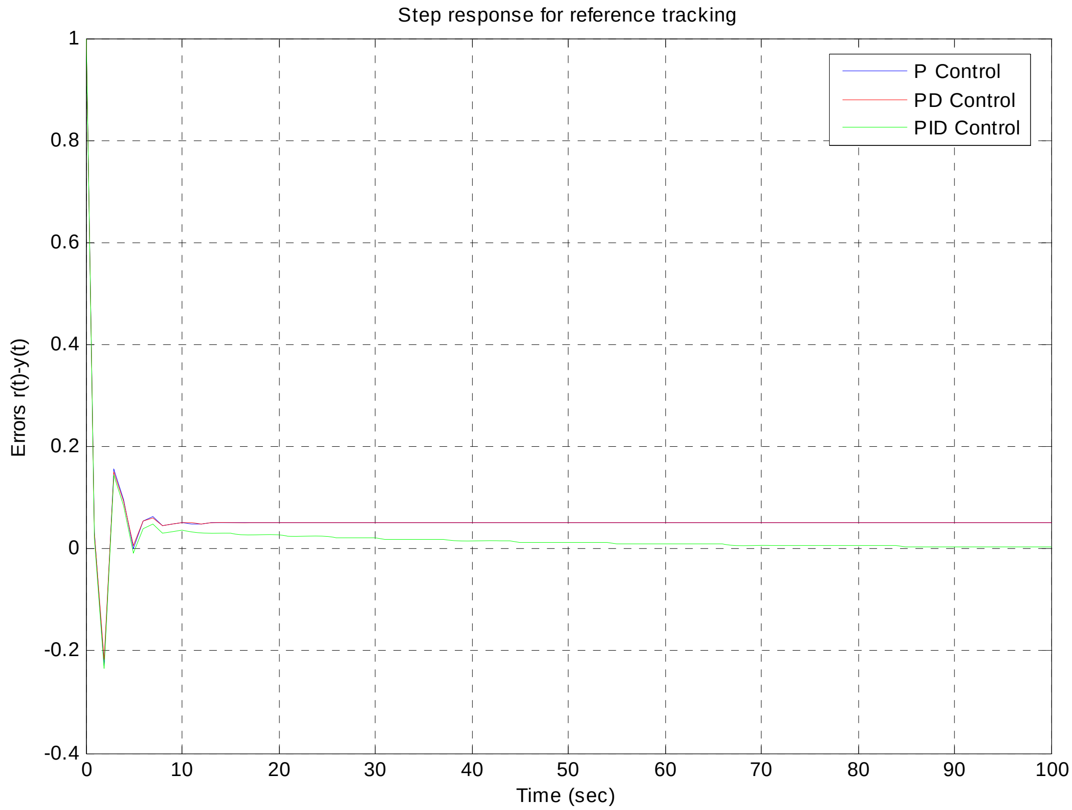
The followings are the plots of responses of $e(t)$. Note the steady state errors,

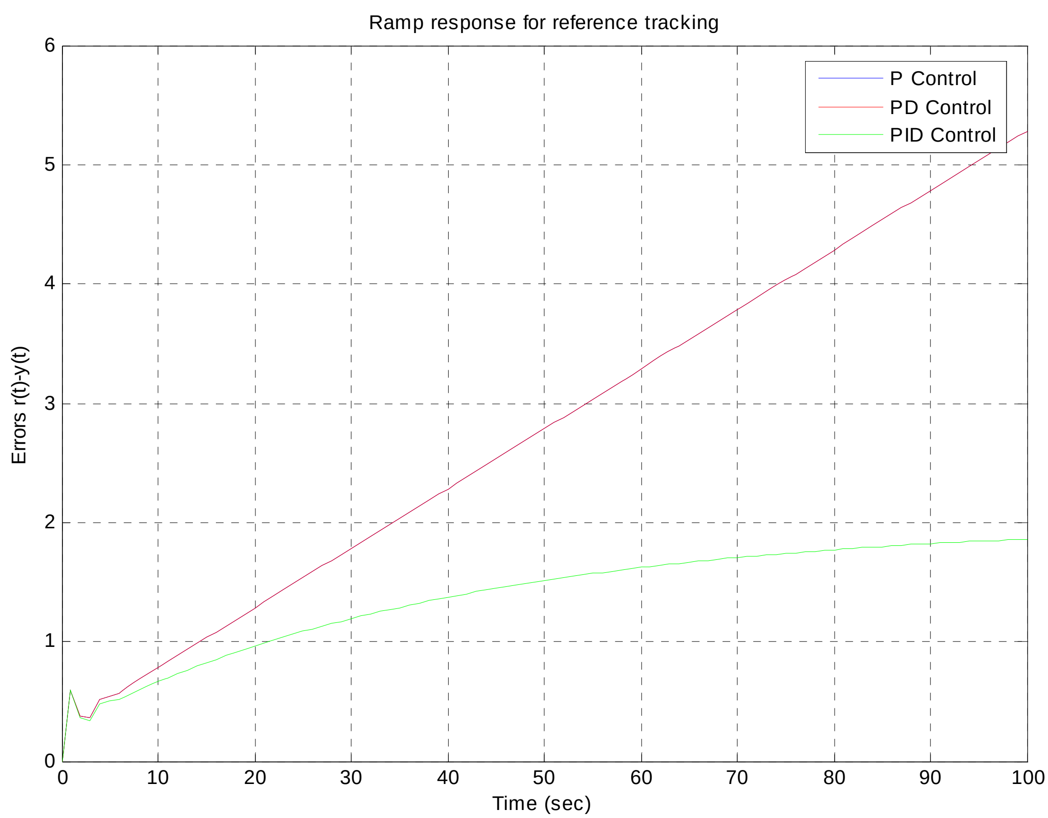
Reference Tracking:

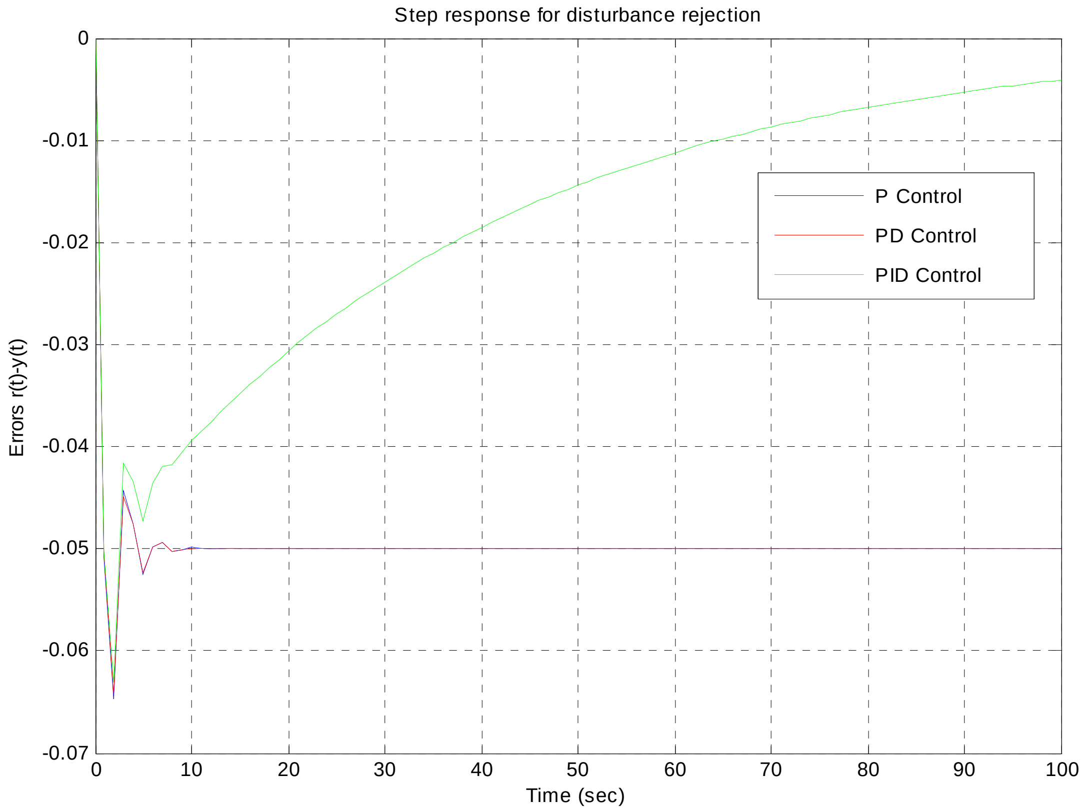
| r(t) | P | PD | PID |
|---|---|---|---|
| 1(t) | 0.05 | 0.05 | 0 |
| t1(t) | ∞ | ∞ | 2 |

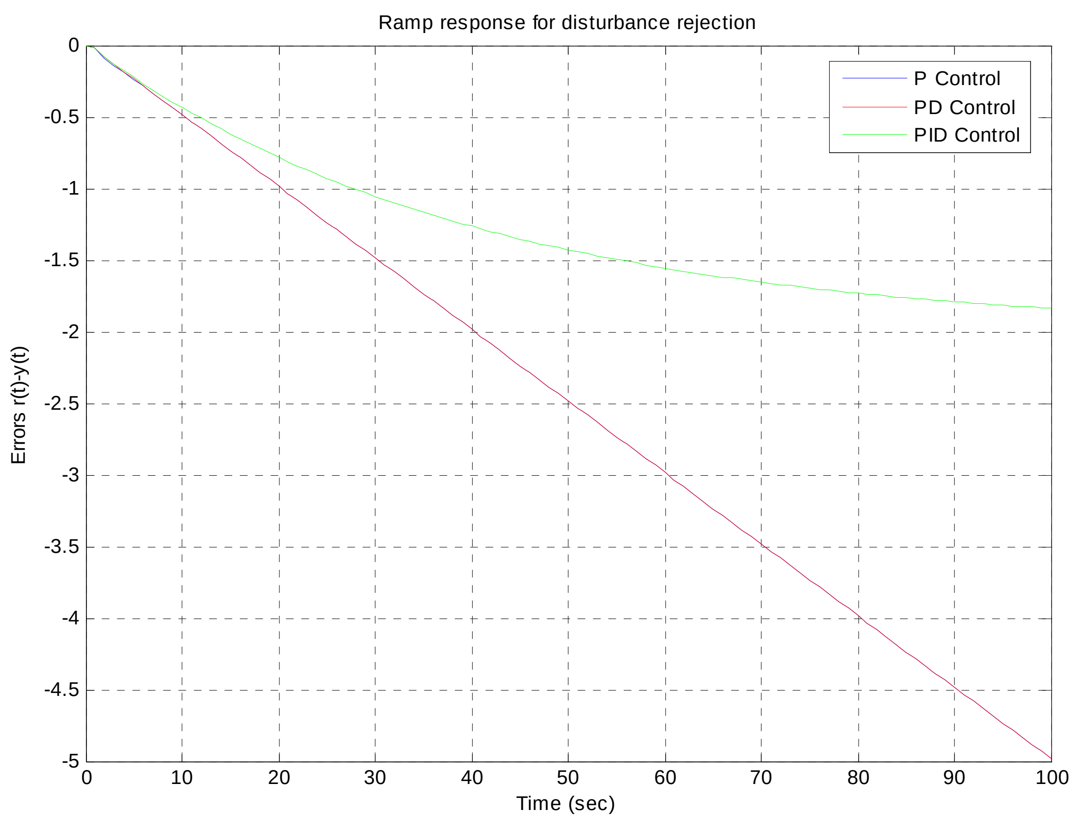
Disturbance Rejection:

| r(t) | P | PD | PID |
|---|---|---|---|
| 1(t) | –0.05 | –0.05 | 0 |
| t1(t) | ∞ | ∞ | –2 |


Step response for reference tracking
Errors r(t)-y(t)
Time (sec)
P Control
PD Control
PID Control
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
10
20
30
40
50
60
70
80
90
100


Ramp response for reference tracking
Errors r(t)-y(t)
Time (sec)
P Control
PD Control
PID Control
6
5
4
3
2
1
0
0
10
20
30
40
50
60
70
80
90
100


Step response for disturbance rejection
Errors r(t)-y(t)
Time (sec)
P Control
PD Control
PID Control
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
0
10
20
30
40
50
60
70
80
90
100


Ramp response for disturbance rejection
Errors r(t)-y(t)
Time (sec)
P Control
PD Control
PID Control
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
0
10
20
30
40
50
60
70
80
90
100

4.22 The transfer function for the plant in a motor position control is given by

$$G(s) = \frac{A}{s(s+a)}$$

If we were able to select values for both A and a, what would they be to result in a system with $K_v = 20$ and $\zeta = 0.707$?

**Solution**:

$$K_v = \lim_{s\to 0} sD(s)G(s) = \lim_{s\to 0} s\frac{A}{s(s+a)} = \frac{A}{a} = 20 \qquad \Rightarrow \qquad A = 20a$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{A}{s^2+as+A} = \frac{A}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad \Rightarrow \qquad \omega_n^2 = A, \quad a = 2\zeta\omega_n$$

$$\Rightarrow \qquad \zeta = \frac{a}{2\sqrt{A}} = 0.707$$

Thus, $a = 40$, $A = 800$.