16.513 Control Systems  --  Final Exam (Spring 2007)

There are 5 problems (Total 100)

1. (15pts) For the differential equation

\[
\dot{x} = \begin{bmatrix} 1 & -2 \\ 5 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u; \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x, \text{ with } x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ u(t)=0, \text{ what is } y(t) \text{ for } t > 0?
\]

2. (15pts) Consider the following system

\[
\dot{x} = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 1 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix} x + \begin{bmatrix} 1 & a \\ 1-a & b \\ -b & 1 \end{bmatrix} u
\]

\[
y = \begin{bmatrix} 1 & a & b & 1 \\ a & 1 & 1 & a \end{bmatrix} x
\]

1) Under what condition on \(\lambda_1, \lambda_2, a, \) and \(b\) is the system controllable?
2) Under what condition on \(\lambda_1, \lambda_2, a, \) and \(b\) is the system observable?

3. (20pts) Perform controllability decomposition on the following system

\[
\dot{x} = \begin{bmatrix} -1 & -2 & -2 & 2 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} -2 & -1 \\ 1 & 1 \\ 0 & 0 \\ 2 & 1 \end{bmatrix} u
\]

\[
y = \begin{bmatrix} 2 & 3 & 0 & 1 \end{bmatrix} x
\]

Is the system stabilizable?
4. (20) For the system

\[
\dot{x} = Ax + Bu = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & 2 & 0 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u, \quad y = Cx = \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} x
\]

Construct an observer gain \( L \) so that the eigenvalues of \( A-LC \) are \(-5 \pm j5, -10 \pm j10\). Construct state feedback gain \( K \) so that the eigenvalues of \( A-BK \) are \(-2 \pm j2, -4 \pm j4\), and each element of \( K \) has absolute value less than 25. Build a simulink model to simulate the closed-loop system with \( u = -Kx_e \), where \( x_e \) is the estimated state. Assume \( x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, x_e(0) = 0 \). Plot the state \( x(t) \), the estimation error \( e(t) = x(t) - x_e(t) \) and the output \( y(t) \). Choose appropriate sampling time so that the curves are smooth. Choose appropriate total simulation time so that a steady state has been reached.

5. (30) Consider the following system, where \( u \) is the control input, \( d \) the disturbance and \( y \) the output. Design a robust tracking and disturbance rejection strategy so that the output \( y \) follows step reference signals. Choose state feedback gain such that the closed-loop system has poles at \(-2 \pm j2, -4 \pm j4, -8\).

\[
\frac{s + 1}{s^4 + s^2 + s - 2}
\]

Build a simulink model to simulate the output response for the following cases:

1. The reference signal \( r(t) = \begin{cases} 0, & t < 0 \\ 2, & 0 \leq t < 7.5 \\ 1, & 7.5 \leq t \end{cases} \); The disturbance \( d(t) = 0 \)

2. The reference signal \( r(t) = \begin{cases} 0, & t < 0 \\ 2, & t \geq 0 \end{cases} \); The disturbance \( d(t) = \begin{cases} 0, & t < 5 \\ 100, & 5 \leq t < 10 \\ 200, & 10 \leq t \end{cases} \)

Plot \( y(t) \) and \( u(t) \) for each case. Print the simulink model. Run simulation long enough so that a steady state has been reached. Pick suitable sampling time so that the curves are smooth. Assume 0 initial condition for the state.