16.513 Control Systems -- Final Exam (Spring 2007)

There are 5 problems (Total 100)

1. (15pts) For the differential equation

$$\dot{x} = \begin{bmatrix} 1 & -2 \\ 5 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u;\ y = \begin{bmatrix} 1 & 1 \end{bmatrix} x,\ \text{with}\ x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ u(t)=0,\ \text{what is } y(t) \text{ for } t > 0?$$

2. (15pts) Consider the following system

$$\dot{x} = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 1 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix} x + \begin{bmatrix} 1 & a \\ 1 & -a \\ b & 1 \\ -b & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & a & b & 1 \\ a & 1 & 1 & a \end{bmatrix} x$$

   1) Under what condition on λ_1, λ_2, a and b is the system controllable?
   2) Under what condition on λ_1, λ_2, a and b is the system observable?

3. (20pts) Perform controllability decomposition on the following system

$$\dot{x} = \begin{bmatrix} -1 & -2 & -2 & 2 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} -2 & -1 \\ 1 & 1 \\ 0 & 0 \\ 2 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 3 & 0 & 1 \end{bmatrix} x$$

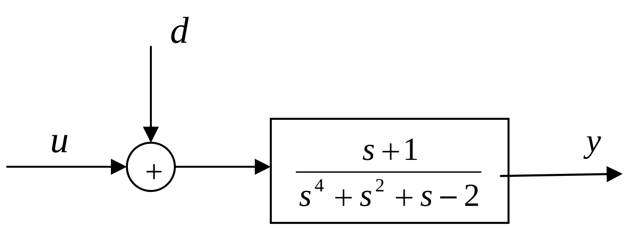
   Is the system stabilizable?

4. (20) For the system

$$\dot{x} = Ax + Bu = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & 2 & 0 & 4 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u, \quad y = Cx = \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} x$$

Construct an observer gain L so that the eigenvalues of A-LC are $-5 \pm j5, -10 \pm j10$. Construct state feedback gain K so that the eigenvalues of A-BK are $-2 \pm j2, -4 \pm j4$, and each element of K has absolute value less than 25. Build a simulink model to simulate the closed-loop system with $u = -Kx_e$, where x_e is the estimated state. Assume $x(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $x_e(0) = 0$. Plot the state x(t), the estimation error $e(t) = x(t) - x_e(t)$ and the output $y(t)$. Choose appropriate sampling time so that the curves are smooth. Choose appropriate total simulation time so that a steady state has been reached.

5. (30) Consider the following system, where u is the control input, d the disturbance and y the output. Design a robust tracking and disturbance rejection strategy so that the output y follows step reference signals. Choose state feedback gain such that the closed-loop system has poles at $-2 \pm j2, -4 \pm j4, -8$.



Build a simulink model to simulate the output response for the following cases:

1. The reference signal $r(t) = \begin{cases} 0, & t < 0 \\ 2, & 0 \le t < 7.5 \\ 1, & 7.5 \le t \end{cases}$; The disturbance $d(t) = 0$
2. The reference signal $r(t) = \begin{cases} 0, & t < 0 \\ 2, & t \ge 0 \end{cases}$; The disturbance $d(t) = \begin{cases} 0, & t < 5 \\ 100, & 5 \le t < 10 \\ 200, & 10 \le t \end{cases}$

Plot $y(t)$ and $u(t)$ for each case. Print the simulink model. Run simulation long enough so that a steady state has been reached. Pick suitable sampling time so that the curves are smooth. Assume 0 initial condition for the state.